Multipoint Flux Finite Element Methods for the Dual-Porosity Darcy Flow Model and Their Efficient Multigrid Solution

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Abstract: The aim of this work is two-fold. On the one hand, we formulate multipoint flux mixed finite element approximation schemes for the single-phase Darcy flow model in dual-porosity rigid porous media. Two classes of methods are derived for quadrilateral elements, depending on the type of meshes (namely, smooth or distorted) under consideration. These are based on the application of symmetric and non-symmetric variants of certain quadrature rules, which were introduced in [1] and [2], respectively, for the single-porosity case.

On the other hand, we focus on the efficient solution of the algebraic systems that arise when considering the aforementioned discretization. Such a task is a crucial part of the numerical simulation process, since it consumes a considerable amount of time in practical applications. The proposed solver is a cell-centered geometric multigrid method [3] on logically rectangular meshes. Remarkably, these meshes take advantage of recent computer architectures that achieve their best performance when structured data are used.

We present a series of numerical experiments that illustrate the convergence behaviour of the approximation schemes, and show the robust performance of the multigrid solver with respect to the permeability tensor coefficient, the mesh size and several families of quadrilateral meshes considered in this work.

References:

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[3] U. Trottenberg, C.W. Oosterlee and A. Schüller. Multigrid. Academic Press, San Diego (2001).