Existence of strange non-chaotic attractors in a quasi-periodically forced piecewise-linear map

Authors:

- Àngel Jorba, Universitat de Barcelona, Spain (angel@maia.ub.es)
- Joan Carles Tatjer, Universitat de Barcelona, Spain (jcarles@maia.ub.es)
- Yuan Zhang, Shanxi University, China (zhangyuan@sxu.edu.cn)

Abstract: We consider the following family of one-dimensional quasiperiodically forced maps:

$$F_{a,b}(x,\theta) = (f_{a,b}(x,\theta), \theta + \omega \text{ mod } 2\pi),$$

where x is real, and θ is an angle. We assume that the second component of the map is an irrational rotation and

$$f_{a,b}(x,\theta) = h_a(x) + b\sin\theta,$$

where $h_a(x) = ax$ if $x \in [-\pi/(2a), \pi/(2a)]$, $h_a(x) = -\pi/2$ if $x \leq \pi/(2a)$ and $h_a(x) = \pi/2$ The dynamics depends on two parameters: b is real and $a > 0$. For this family we can prove the existence of both smooth and non-smooth pitchfork bifurcations. As what happens for a smooth pitchfork bifurcation, for $a > 1$ there exists $b = b_a(a)$ such that for $b < b_0(a)$ $f_{a,b}$ has two continuous attracting invariant curves and one continuous repelling invariant curve, and for $b > b_0(a)$ there is only one continuous attracting invariant curve. But for $b = b_0$ the situation is different: we have two non-continuous attracting invariant curves and one continuous repelling invariant curve such that the attracting invariant curves intersect the repelling one in a zero-Lebesgue mesure set of angles. We can say that in this case we have a non-chaotic strange attractor, consisting on the closure of the three invariant curves (see [?]).

It is remarkable that this family is a simplification of the smooth family $G_{a,b}(x,\theta) = (\arctan(ax) + b\sin(\theta), \theta + \omega)$ for which there is numerical evidence of existence of non-smooth pitchfork bifurcations (see [?]).

References:

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