Integrability criteria for nonlinear oscillators and suprintegrable geodesic flows

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Abstract:

We consider a family of cubic, with respect to the first derivative, second-order differential equations. This family of equations is a projection of equations for the geodesics of a two-dimensional Riemannian manifold and, therefore, these equations are often called projective.

The family of projective equations is closed with respect to nonlocal transformations. This allow one to study equivalence problems for this family and its integrable members, for example linear or Painlevé-type equations. We demonstrate that solutions to these equivalence problems lead to new integrability criteria for the projective equations [1, 2, 3]. For each member of such equivalence classes, it is possible to obtain a first integral and integrating factor, and, in the case of autonomous equations and transformations, invariant curves.

First integrals of projective equations can be lifted to the first integrals of the corresponding Hamiltonian systems for geodesics, if a certain projective equation corresponds to a metric. We propose an approach to construct superintegrable metrics using non-autonomous first integrals of autonomous projective equations [4]. We suppose that the Hamiltonian system for geodesics admits a linear with respect to momentum first integral and classify all autonomous projective equations that correspond to such a metric. We demonstrate that all these equations either can be linearized by nonlocal transformation or trivially integrable.

We use this approach to connect several applied nonlinear oscillators with superintegrable Riemannian metrics and explicitly construct their additional first integrals, which can be polynomial, rational and transcendental function in momenta. As examples of nonlinear oscillators we consider the anharmonic oscillator, the cubic Liénard oscillator with linear damping, the Kolmogorov system and cubic oscillators with biological applications.

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