A Trefftz discontinuous Galerkin method for the acoustic scattering in a waveguide

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Abstract: We simulate the propagation of time-harmonic waves along an unbounded tubular waveguide by means of a Trefftz Discontinuous Galerkin (TDG) formulation which we discretize with the superposition of Plane Waves (PWDG).

We first rewrite the problem on a bounded computational domain by using the Neumann-to-Dirichlet map on the truncation walls. We formulate variationally this problem in a DG way, in the sense that the interelement continuity is imposed weakly within the variational formulation by introducing suitable numerical fluxes: Indeed, we make use of standard numerical fluxes except for facets on the truncation boundary, for which we use some more exotic numerical fluxes. We then get a consistent and coercive formulation which achieves quasi-optimal convergence when discretized with e.g. plane waves.

For the PWDG formulation in two dimensions, we verify and illustrate the behavior of the numerical solutions and their order of convergence with numerical experiments obtained with a Python code that we have developed from scratch. In particular, we investigate the role of the choice of the flux parameters, as well as the instability and ill-conditioning inherent in this kind of methods.

References:

[1] R. Hiptmair, A. Moiola, I. Perugia, *A survey of Trefftz methods for the Helmholtz equation*, In: Building Bridges: Connections and Challenges in Modern Approaches to Numerical Partial Differential Equations, Springer Lect. Notes Comput. Sci. Eng., edited by G.R. Barrenechea, F. Brezzi, A. Cangiani, E.H. Georgoulis, 2016, pp. 237–278

[2] S. Kapita, P. Monk, V. Selgas, *A Trefftz Discontinuous Galerkin method for time-harmonic waves with a generalized impedance boundary condition*, Applicable Analysis 99(3), 2020, pp. 379–40