Existence for quasilinear elliptic equations involving asymptotic linear growth operators

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Abstract: This talk is devoted to study existence of solutions to problems

$$\begin{cases} -\text{div } (\phi(|\nabla u|)\nabla u) = f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded open set having Lipschitz continuous boundary, $N \geq 2$ and $f \in L^{N,\infty}(\Omega)$. Regarding the function $\phi : \mathbb{R}_+ \to \mathbb{R}_+$, it satisfies

(ϕ_1) Function $s \mapsto \phi(s)s$ is non-decreasing in $\mathbb{R}_+$;

(ϕ_2) $\lim_{s\to+\infty} \phi(s)s = 1$;

(ϕ_3) Function $s \mapsto \phi(s)s$ is continuous in $(0, +\infty)$.

Main examples of equations that can be writen as in (1) for suitable functions ϕ are:

1. When $\phi(s)s = 1$ for $s > 0$, the operator in (1) becomes the 1-Laplacian.
2. When $\phi(s)s = \frac{s}{\sqrt{1+s^2}}$, we find the prescribed mean curvature operator.

It is well–known that in both cases a smallness condition on the datum f is necessary to obtain existence of solution. Our aim is to identify, in the general case (1), the threshold on the size of the datum so that below this value there exists a solution and above it there does not.

Formally, problem (1) is the Euler-Lagrange equation of minimizing an asymptotic linear growth functional.