Numerical approximation of PGD modes to parameterized elliptic problems

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Abstract: The Proper Generalized Decomposition (PGD) is a reduced order method to solve parameterized differential equations. This method compute approximations of the solution under the form of a low-rank tensorized decomposition and so that, these approximations are constructed without any information on the solution, but only on the differential equation to be solved.

This work focus on the analysis of the aproximation of the PGD expansion modes for the parameterized solutions of elliptic equations. The modes are obtained by alternated Galerkin problems in a Lebesgue space of the parameter set and the space in which lives the solution of the equation. In practice, both problems have to be discretized to be resolved and so, the computation of the modes requires a double discretization. On one hand, we approximate the integral on the parameter set by means of composed quadrature formulas based upon finite element approximations of the integrands with respect to the parameter. On the other hand, the framework space for the solution is approximated by a finite-dimensional subspace. In this context, we prove that the discrete PGD modes converge to the continuous ones in the mean quadratic norm associated to the parametric elliptic operator. For that, we are based on the casting of the modes PGD as solutions of a calculus of variations problem on optimal subspaces of finite dimension that minimize the residual in this mean quadratic norm. We have also performed some numerical experiments to asses the convergence of the PGD modes and to provide some indications on the rate of convergence. The results show an spectral or algebraic convergence according to the regularity of the solution. Moreover, in all the cases we have observed a spectral convergence of the truncated PGD series in the parametric L^2 norm.