Blow-up estimates and a priori bounds for the positive solutions of a class of superlinear indefinite elliptic problems

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Abstract: In this talk we present some new blow-up estimates for the positive explosive solutions of a paradigmatic class of elliptic boundary value problems of superlinear indefinite type:

$$\begin{cases} \mathscr{L}u = \lambda u + a(x)u^r & \text{in } \Omega, \\ \mathscr{B}u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain of $\mathbb{R}^N$, $N \geq 1$, of class $\mathcal{C}^2$, $\lambda \geq 0$,

$$\mathscr{L}u = -\mathrm{div}(A(x)\nabla u),$$

uniformly elliptic in Ω and $\mathfrak{B}$ is any boundary operator of non- classical mixed type on $\partial\Omega$. These estimates are obtained by combining the scaling technique of Guidas–Spruck [2] together with a generalized De Giorgi–Moser weak Harnack inequality found, very recently, by Sirakov [4, 5]. In a further step, based on a comparison result of Amann and López-Gómez [1], we will show how these bounds provide us with some sharp a priori estimates for the classical positive solutions of (1). It turns out that this is the first general result where the decay rates of the potential $a(x)$ do not play any role for getting a priori bounds for the positive solutions when $N \geq 3$. This is a joint work with J. López-Gómez [3].

References:

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