Bidiagonal decompositions of totally positive Gram matrices and applications

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Abstract:

Finding classes of matrices with relevant applications, for which linear algebraic computations can be efficiently performed to high relative accuracy, is an important research topic. Bidiagonal decompositions of totally positive matrices, by means of Neville elimination, have become an instrumental tool to parameterize and perform accurate computations involving these matrices.

Gram matrices are implicit in many diverse applications, such as the finite element method, the model fitting of the covariance structure, machine learning (see [1], [2], [3]) and some of the fundamental problems of interpolation and approximation which lead to interesting related linear algebra computations. Unfortunately, Gram matrices are often ill-conditioned and, therefore, computations with these matrices can lose accuracy as the dimension of the problem increases.

Hilbert matrices $H_n = (1/(i+j-1))_{1\leq i,j,\leq n+1}$ are well-known notoriously ill-conditioned Gram matrices corresponding to monomial polynomial bases with respect to the usual inner product

$$< f, g >= \int_0^1 f(t)g(t)\,dt.$$

In [4, 5], it is shown that these matrices are strictly totally positive and a bidiagonal decomposition to Hilbert matrices is obtained providing matrix computations to high relative accuracy.

In this talk new contributions to this field will be presented (see [6, 7, 8, 9]). In particular, some examples of Gram matrices of totally positive polynomial bases with respect to several inner products will be considered. Their total positivity will be analyzed and a bidiagonal decomposition of the considered matrices will be derived. Furthermore, using the proposed representations, it will be shown the numerical resolution of linear algebra problems with these matrices is achieved to high relative accuracy.

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