Exponential small splitting of separatrices in a molecular model

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Abstract:

In this presentation, we will study the dynamics of the hydrogen atom interacting with a circularly polarized microwave field. This problem can be effectively modeled as a perturbed Kepler problem, which, upon suitable transformations of coordinates and time, manifests as a Hamiltonian system with two degrees of freedom, dependent on a single parameter ($K > 0$). Its formulations is given by

$$H(x, y, p_x, p_y) = \frac{1}{2}(p_x^2 + p_y^2) - xp_y + yp_x - \frac{1}{r} + Kx. \tag{1}$$

Despite the simplicity of the model, which can be seen as the simplest perturbation of the Kepler problem, its dynamics exhibit great complexity. The system of ordinary differential equations associated with the Hamiltonian H has properties of interest for our study. In particular, it has two equilibrium points, L_i, located at $(x_{L_i}, 0)$, $i = 1, 2$ with $x_{L_1} < 0$ and $x_{L_2} > 0$. L_1 is a center-saddle for any $K > 0$ and L_2 is a center-center for $K \leq K_{\text{crit}}$ and a complex saddle for $K > K_{\text{crit}}$ where $K_{\text{crit}} = 3^{-4/3}/2$. In this talk, we will mainly focus on the behavior of the invariant manifolds associated with the equilibrium point L_1.