RK integrators for the DG discretization of linear Friedrichs systems

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Abstract: Symmetric Friedrichs systems include a broad category of linear hyperbolic systems that cover various mathematical models for linear wave propagation phenomena. The discontinuous Galerkin method stands out as a favored technique for spatial discretization in these models, offering greater versatility compared other methods such as finite elements. By employing off-centered numerical fluxes, it introduces artificial dissipation, which proves useful in controlling spurious oscillations observed with centered fluxes, particularly when modeling convection phenomena.

In time-dependent simulations, the temporal discretization of the semidiscrete problem derived from the discontinuous Galerkin method, in both conservative and dissipative scenarios, is crucial. For large-scale simulations, explicit schemes are preferred, raising concerns about their stability.

In our presentation, we will discuss various results regarding the stability of RK schemes by means of the energy method continuing and (in some aspects) extending the works in [1,2].

References:

- E. Burman and A. Ern and M. A. Fernández. Explicit Runge-Kutta schemes and finite elements with symmetric stabilization for firstorder linear PDE systems. SIAM Journal on Numerical Analysis, 48(6):2019–2042, 2010.
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