A numerical method to solve Maxwell's equations in singular 3D geometry

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Abstract: We propose a variational method to compute the three-dimensional Maxwell equations in an axisymmetric singular domain generated by the rotation of a singular polygon around one of its sides, which contains reentrant corner or edges. Due to the axisymmetric assumption, the singular computational domain reduces to a subset of R^2. However, the electromagnetic field and other vector quantities still belong to R^3.

Taking advantage of the fact that the domain becomes two-dimensional, through Fourier analysis in the third dimension, one arrives to a sequence of singular problems set in a 2D domain, depending on the Fourier variable k. Under these conditions, the 3D solution is solved by addressing several 2D problems, each dependent on k.

Furthermore, for each mode k, we can show that the solution can be decomposed into a regular and a singular part. Therefore, the regular part can be computed using a classical finite element method. The singular part belongs to a finite-dimensional subspace, its dimension being equal to the number of reentrant corners and edges of the 2D polygon that generates the 3D domain.

We first compute this singular part using an *ad hoc* numerical method only for $k = 0, \pm 1, 2$, where the mode $k = 2$ appearing as a "stabilization" mode for all other k. Then, the total solution will be computed based on a non-stationary variational formulation. Numerical examples will be presented to illustrate that the proposed method can capture the singular part of the solution. This approach can also be viewed as the generalization of the Singular Complement Method to time-dependent three-dimensional axisymmetric problem [**?**], [**?**].

References:

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