The discrete-time para-Hermitian rational eigenvalue problem

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Abstract:

Strongly minimal linearizations of rational matrices [1] are linear polynomial system matrices of the form:

$$L(z) := \begin{bmatrix} zA_1 - A_0 & zB_1 - B_0 \\ -zC_1 + C_0 & zD_1 - D_0 \end{bmatrix},$$

where $zA_1 - A_0$ is regular, and the pencils $\begin{bmatrix} zA_1 - A_0 & zB_1 - B_0 \end{bmatrix}$ and $\begin{bmatrix} zA_1 - A_0 \\ -zC_1 + C_0 \end{bmatrix}$ have no finite or infinite eigenvalues. Strongly minimal linearizations contain the complete information about the zeros, poles and minimal indices of the corresponding rational matrix $R(z)$ and allow to recover very easily its eigenvectors and minimal bases. Therefore, they can be combined with algorithms for the generalized eigenvalue problem for computing the complete spectral information of $R(z)$.

Rational matrices play a fundamental role in systems and control theory [2], and in some important applications they can have a particular self-conjugate structure. In [1] it is shown how to construct strongly minimal linearizations that preserve it for Hermitian and skew-Hermitian rational matrices, with respect to the real line, and for para-Hermitian and para-skew-Hermitian rational matrices, with respect to the imaginary axis. In this talk we describe the solution for discrete-time para-Hermitian rational matrices. These are rational matrices that are para-Hermitian on the unit circle. However, a linearization can not be discrete-time para-Hermitian. Therefore, given a discrete-time para-Hermitian rational matrix $R(z)$, we instead construct a palindromic linearization for $(1+z)R(z)$, whose eigenvalues that are not on the unit circle preserve the symmetries of the zeros and poles of $R(z)$. This can be solved via Möbius transformations. We also give a constructive method that is based on an additive decomposition into the stable and anti-stable parts of $R(z)$.

References:

[1] F. M. Dopico, M. C. Quintana, P. Van Dooren, *Strongly minimal self-conjugate linearizations for polynomial and rational matrices*, SIAM J. matrix anal. appl., Vol.43 (3), (2022) 1354–1381.

[2] H. H. Rosenbrock, *State-space and Multivariable Theory,* Thomas Nelson and Sons, London, 1970.