Semi-Analytical Solutions for Systems with Slowly Varying Parameters

Authors:

- Felipe Ponce-Vanegas, Basque Center for Applied Mathematics (fponce@bcamath.org)
- András Bártfai, Department of Applied Mechanics, Budapest University of Technology and Economics, Hungary (andras.bartfai@mm.bme.hu)
- Zoltán Dombóvári, Department of Applied Mechanics, Budapest University of Technology and Economics, Hungary (dombovari@mm.bme.hu)

Abstract: The central equation in the analysis of vibratory systems in engineering is

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = F.$$

A large body of literature has been developed around the approximation that the mass M, damping C, and stiffness K matrices are constants, which makes the analysis more tractable.

This approximation is untenable for many processes in industrial environments, and their study mainly relies on numerical simulations, which may not only obscure the structure of the solutions but could also misrepresent the system. In particular, when there are two processes, one fast and another slow — for example, a turning process with a beam rotating at high speed and slowly varying length — it may occur that at bifurcation points, the approximated solution computed by traditional numerical schemes, like the Runge–Kutta methods, is highly inaccurate due to the computer's inability to faithfully represents numbers like $1 + 10^{-100}$, which leads to significant divergences from the real solution in a short period of time [2].

We propose a semi-analytical method for solving scalar systems, but its theoretical correctness hangs upon a conjecture that we call the Existence of Steady State Conjecture [1]. We explain the reason for the strong instability of these systems, and why we believe that a delayed loss of stability observed in other numerical studies [3] will not play any role in real-world systems.

References:

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[3] Baer, S., Erneux, T., and Rinzel, J. (1989) The slow passage through a Hopf bifurcation: delay memory effects, and resonance. *SIAM J. Appl. Math.*, 49(1):55-71.