Nonlinear Fractional Schrödinger Equations coupled by power–type nonlinearities

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Abstract: In this work we study the following class of systems of coupled nonlinear fractional Schrödinger equations,

$$\begin{cases} (-\Delta)^s u_1 + \lambda_1 u_1 = \mu_1 |u_1|^{2p-2} u_1 + \beta |u_2|^p |u_1|^{p-2} u_1 & \text{in } \mathbb{R}^N, \\ (-\Delta)^s u_2 + \lambda_2 u_2 = \mu_2 |u_2|^{2p-2} u_2 + \beta |u_1|^p |u_2|^{p-2} u_2 & \text{in } \mathbb{R}^N, \end{cases}$$

where u_1, $u_2 \in W^{s,2}(\mathbb{R}^N)$, with $N = 1, 2, 3$; λ_j, $\mu_j > 0$, $j = 1, 2$, $\beta \in \mathbb{R}$, $p \geq 2$ and $\dfrac{p-1}{2p} N < s < 1$. We prove the existence of positive radial bound and ground state solutions provided the parameters $\beta, p, \lambda_j, \mu_j$, $(j = 1, 2)$ satisfy appropriate conditions. We also study the previous system with m-equations,

$$(-\Delta)^s u_j + \lambda_j u_j = \mu_j |u_j|^{2p-2} u_j + \sum_{\substack{k=1 \\ k \neq j}}^{m} \beta_{jk} |u_k|^p |u_j|^{p-2} u_j,$$

where λ_j, $\mu_j > 0$ for $j = 1, \ldots, m \geq 3$, the coupling parameters $\beta_{jk} = \beta_{kj} \in \mathbb{R}$ for $j, k = 1, \ldots, m$, $j \neq k$. We prove similar results as for $m = 2$, depending on the values of the parameters $p, \beta_{jk}, \lambda_j, \mu_j$.