Global effects of a saturation term in a heterogenous predator-prey model

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Abstract: In this talk we analyze the spatially heterogeneous predator-prey model

$$\begin{cases} \mathfrak{L}_1 u = \lambda u - a(x)u^2 - b(x)\dfrac{uv}{1+\gamma m(x)u} & \text{in } \Omega, \\ \mathfrak{L}_2 v = \mu v + c(x)\dfrac{uv}{1+\gamma m(x)u} - d(x)v^2 & \text{in } \Omega, \\ \mathfrak{B}_1 u = \mathfrak{B}_2 v = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\mathfrak{L}_1$ and $\mathfrak{L}_2$ are second order uniformly elliptic operators, and $\mathfrak{B}_1$ and $\mathfrak{B}_2$ are general boundary operators of mixed type. In (**??**), $a, d > 0$, $b, c \gneq 0$, $\gamma > 0$ and $m \geq 0$ in $\bar{\Omega}$, while $\lambda, \mu \in \mathbb{R}$ are the bifurcation parameters. The term $m(x)$ measures the level of saturation of the predator at any particular location $x \in \Omega$ where $m(x) > 0$ (Holling-Tanner response), while saturation effects do not play any role if $m(x) = 0$ (Lotka-Volterra response).

During the talk, they will be first ascertained the regions in the plane (λ, μ) in which coexistence states exist or could exist. Then, considering a shadow system appearing when $\gamma \uparrow +\infty$, it will be provided a generic multiplicity result ensuring the existence of, at least, two coexistence states of (**??**) for γ large enough in the region in which one of the semitrivial positive solutions is linearly stable. Moreover, in some special cases, a S-shaped component appears implying the existence of, at least, three coexistence states. Finally, when the amplitude of $m(x)$ is small enough, we will see that there is uniqueness in the one-dimensional counterpart of (**??**).

References:

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