Heteroclinic connections around double resonances in an Arnold model

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Abstract: In his seminal paper (see [1]), Arnold introduced the concept of Arnold diffusion for nearly integrable systems: the existence of trajectories presenting an arbitrarily large drift in the space of actions due to the accumulated effect of an arbitrarily small perturbation over a large period of time. He designed a very particular model where he could prove its existence and raised the question of whether it occurs generally in nearly integrable systems.

An important technical difficulty when trying to answer this question is the appearance of exponentially small phenomena. Arnold dealt with it by introducing two independent perturbative parameters, one of which was exponentially small with respect to the other. This is convenient to illustrate the existence of diffusion, but it creates a very artificial setting that does not include the physically relevant nearly integrable systems. Hence, this technique does not provide a satisfactory answer.

In this work we analyze a generalized Arnold model in the more natural setting where both parameters are of the same order. This means that we need to apply techniques of analysis of exponentially small phenomena. We prove the existence of heteroclinic connections of order $\mathcal{O}(|\mu|\sqrt{\varepsilon})$ —μ and ε are the two parameters in the system—, although we cannot chain them to obtain global diffusion. Our approach involves averaging the system around double resonances and studying separately the averaged and the complete systems. We find that the averaged system presents heteroclinic connections which persist in the complete system.

References:

[1] V.I. Arnold. *Instability of dynamical systems with several degrees of freedom*. Sov. Math. Doklady (1964), 5:581–585.

[2] David Sauzin. *A new method for measuring the splitting of invariant manifolds*. Annales scientifiques de l'École Normale Supérieure, Série 4, Tome 34 (2001) no. 2 159-221.