Approximate Inverse LU preconditioning applied to least squares problems

Authors:

- José Marín, Universitat Politècnica de València (jmarinma@imm.upv.es)
- José Mas, Universitat Politècnica de València (jmasm@imm.upv.es)

Abstract:

In this work we consider the application of approximate inverse LU preconditioners to compute preconditioners for the iterative solution of sparse least squares problems of the form

$$\min_x \|b - Ax\|_2, \tag{1}$$

where $A \in \mathbb{R}^{m \times n}$ $(m \geq n)$ is a large and sparse matrix with full column rank. We consider the solution of (1) with the preconditioned CGLS method, [3] which implicitly applies the conjugate gradient method to the normal equations

In this work we apply the V–AISM preconditioner introduced in [2] which is a variant of the AISM preconditioner [1]. The main difference is that the Sherman-Morrison formula is applied multiplicatively that allows for a compact representation of the partial factors. The results of numerical experiments show that this new preconditioner is efficient compared with other approximate inverse preconditioners that appear in the bibliography.

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