Unexpected results for a singular elliptic problem

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Abstract

In this lecture, I will present recent results obtained in collaboration with Daniela Giachetti (Universitá di Roma, Sapienza), Francois Murat (Sorbonne Université, París), and Francesco Petitta (Universitá di Roma, Sapienza), for the one-dimensional singular boundary value problem

$$-\frac{d}{dx}\Big(a(x)\frac{du(x)}{dx}\Big) = -\frac{d\phi(u(x))}{dx} - \frac{dg(x)}{dx} \quad \text{in } (0,L), \quad u(0) = u(L) = 0,$$

where the model for the singular function ϕ is $\phi(s) = \dfrac{1}{|s|^\gamma}$ with $\gamma > 0$.

This singular problem presents a number of unexpected phenomena: nonexistence of solutions under certain assumptions, existence of an infinite number of solutions under other assumptions, and non-continuity of the solution with respect to the data.