Semilinear eigenvalue problems with an unbounded interval of bifurcation points

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Abstract:
In this work, we study the behavior of the set of solutions of the semilinear elliptic problem

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^N$ and f is a nonnegative continuous real function with multiple positive zeros. First, we analyze the set of the solutions whose maximum is between two consecutive positive zeros of f, arriving to the existence of an unbounded continuum of solutions with $\subset$-shape. Then, we study the asymptotic behavior of the countable many unbounded continua in the case in which f has a sequence of positive zeros. For the model cases $f(t) = t^r(1 + \sin t)$ and $f(t) = t^r \left(1 + \sin \frac{1}{t}\right)$ with $r \geq 0$, we show the surprising fact that there are some values of r for which every $\lambda > 0$ is a bifurcation point (either from infinity or from zero) that is not a branching point.

References:

[1] J. Carmona, A. J. Martínez Aparicio, P. J. Martínez-Aparicio. *Intervals of bifurcation points for semilinear elliptic problems*. Preprint.