**New functional inequalities with applications to the Arctan-Fast Diffusion Equation**

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**Abstract:** In this talk, we prove a couple of new nonlinear functional inequalities of Sobolev type akin to the logarithmic Sobolev inequality. In particular, one of the inequalities reads

$$\int_{\mathbb{S}^1} \arctan\left(\frac{\partial_x u}{u}\right) \partial_x u \, dx \geq \arctan\left(\|u(t)\|_{\dot{W}^{1,1}(\mathbb{S}^1)}\right) \|u(t)\|_{\dot{W}^{1,1}(\mathbb{S}^1)}.$$

Then, these inequalities are used in the study of the nonlinear *arctan*-fast diffusion equation

$$\partial_t u - \partial_x \arctan\left(\frac{\partial_x u}{u}\right) = 0.$$

For this highly nonlinear PDE we establish a number of well-posedness results and qualitative properties.