Efficient, robust and accurate method for simulating a tumor growth model

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Abstract: In Sciences and Engineering, there is a diversity of problems to be studied in the area of Computational Fluid Dynamics, which include relevant applications in Biomedicine. For the solution of these problems, numerical methods are employed regardless of complexity, geometry, physical parameters, boundary and initial conditions. However, one of the main disadvantages of numerical methods concerns the determination of computational errors associated with their use, in which numerical solutions can be affected by truncation, iteration and rounding errors. Although numerical errors cannot be completely eliminated, it is essential that they are controlled or minimized. Among the sources of numerical error, discretization error is considered the most significant, requiring careful analysis. Therefore, in this work, we examine the effectiveness of Richardson's Extrapolation as an alternative to decrease and estimate discretization error in the numerical solution of a two-dimensional mathematical model of tumor growth described by a system of four partial differential equations in transient regime, where two of the equations are nonlinear [1]. The numerical approach adopted involves the use of the finite difference method, with central differences for the spatial discretization and the Crank-Nicolson method for the discretizationin time. Linearization is performed by applying Taylor Series expansion, while the solution of the resulting system of linear equations is addressed with a Gauss-Seidel solver together with the multigrid method to accelerate the convergence of the iterative process. Numerical tests were conducted with satisfactory results both for convergence acceleration and for reduction and estimation of discretization error for a problem with a known analytical solution, verifying the robustness and effectiveness of the multigrid method and the accuracy of the used error estimators. Subsequently, numerical tests were conducted with satisfactory results for a realistic problem, ensuring the reliability and accuracy of the results.

References:

[1] M. Kolev and B. Zubik-Kowal. Numerical solutions for a model of tissue invasion and migration of tumour cells. Computational and mathematical methods in medicine, vol. 2011, Article ID 452320, 16 pages, 2011.