A high order ImEx method for the shallow water model

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Abstract: This talk is devoted to the development and analysis of a robust and efficient high order numerical scheme for the shallow water flows in the low Froude number limit. We focus on ocean and coastal simulations at different scales, in particular, on the variation of the Froude number that goes from 1 at the coastline to two or three orders less offshore. In order to propose an efficient method in such regime, a part of the system has to be considered implicitly, leading to an ImEx (Implicit Explicit) scheme. In order to limit the size and number of linear systems to be solved, the CPR scheme [1] is a good first order candidate. The CPR approach is a fully diagonal segregated method which only relies on the implicit treatment of the water height and hybrid mass fluxes using explicit velocities. Concerning the high order in time integration, several Runge-Kutta schemes can be found in the literature [2] in the context of ImEx schemes, however to limit the number of linear systems to solve, we focus on Crank Nicolson schemes. For the space discretization, a classical second order MUSCL reconstruction is used. We finally show, thanks to one-and two-dimensional test cases, that the developed scheme achieves the theoretical second-order rate of convergence. Furthermore, we conduct a comparative analysis of CPU times between the ImEx and explicit schemes, revealing important computational savings with the ImEx scheme particularly under the low Froude regime.

References:

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