Well-distributed points on the sphere and the real projective plane

Authors:

- Carlos Beltrán, Universidad de Cantabria (beltranc@unican.es)
- Ujué Etayo, Universidad de Cantabria (etayomu@unican.es)
- Pedro R. López-Gómez, Universidad de Cantabria (lopezpr@unican.es)

Abstract:

In the last decades, the problem of evenly distributing points on manifolds like spheres and projective spaces has attracted the attention of the mathematical community due to its theoretical interest and its numerous practical applications, constituting nowadays a very active field of research. One of the main open problems in the area is Smale's 7th problem, which asks for a collection of spherical points with very low logarithmic energy, in some precise sense. This problem dates back to 1993, when Michael Shub and Stephen Smale [2] discovered an astounding fact: if the logarithmic energy of a set of N points on the sphere is very close to the minimum, then the monic polynomial associated to the stereographic projection of these points has small condition number.

In this talk, I will tackle the problem of distributing points on the usual two-dimensional sphere and on the real projective plane. More precisely, I will present a generalization of a family of spherical points, the Diamond ensemble [1], containing collections of N points on the sphere with low logarithmic energy for any natural number N. In addition, I will also show how the ideas for distributing points on the sphere can be extended to the real projective plane, thereby obtaining lower and upper bounds for the Green and logarithmic energies which constitute the best results in that regard thus far [3].

References:

[1] C. Beltrán and U. Etayo, *The Diamond ensemble: a constructive set of spherical points with small logarithmic energy*, Journal of Complexity, **59**, 101471, 2020.

[2] M. Shub and S. Smale, *Complexity of Bezout's Theorem III. Condition Number and Packing*, Journal of Complexity, **9**, 1, 4-14, 1993.

[3] C. Beltrán, U. Etayo and P. R. López-Gómez, *Low-energy points on the sphere and the real projective plane*, Journal of Complexity, **76**, 101742, 2023.