Neural Network based Finite Volume Methods for hyperbolic conservation laws

Authors:

- Manuel Jesús Castro Díaz, Universidad de Málaga (mjcastro@uma.es)
- León Miguel Ávila León, Universidad de Málaga (leonavilaleon@uma.es)

Abstract:

The rapid evolution of Physics Informed Neural Networks (PINNs) [1] and Variational PINNs (VPINNs) [2] signifies a transformative shift in the field of computational mathematics, particularly in solving complex non-linear partial differential equations (PDEs). These methodologies efficiently encode physical laws into the architecture of neural networks, allowing the numerical approximation of PDEs.

In this context, we propose to combine PINNs and the traditional finite volume methods (FVM), which are widely used for numerical approximations of hyperbolic conservation laws. First, we consider the integral form of the hyperbolic conservation law and a suitable partition of our computational mesh into cells. Next, we will use neural networks in each cell as a reconstruction operator in the FVM framework. The advantage of using neural networks is to be able to consider implicit methods without increasing the complexity, which allows us to increase stability and to be able to use larger time steps. The neural network performs the crucial task of reconstructing intercell fluxes. The way the method is assembled will allow us to construct well-balanced methods in this framework [3], or to make use of entropy-conservative numerical fluxes.

Moreover, the efficiency of the proposed method will be demonstrated through its application to a variety of hyperbolic conservation laws, including Burgers' equations, shallow water equations, and the compressible Euler equations.

References:

[1] S. Mishra and R. Molinaro, *Estimates on the generalization error of physics-informed neural networks for approximating PDEs*. IMA Journal of Numerical Analysis, 2022.

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[3] M.J. Castro and T. Morales de Luna and C. Parés, *Well-Balanced Schemes and Path-Conservative Numerical Methods*. Elsevier, 2017.