Well-balanced reduced order models based on POD for hyperbolic systems of balance laws

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Abstract: Let us consider a general one-dimensional hyperbolic system of balance laws of the form

$$W_t + F(W)_x = S(W)H_x + R(W), \quad x \in I,\, t > 0, \tag{1}$$

which describe a great number of relevant phenomena in fluid dynamics, such as shallow water models, multiphase flow models, gas dynamic, etc. Systems of balance laws of the form (1), which admit non-trivial stationary solutions satisfying

$$F(W)_x = S(W)H_x + R(W), \tag{2}$$

have been widely applied in simulation of the waves generated by small perturbations of stationary solutions: think, for instance, of tsunami waves in the ocean. The design of well-balanced numerical schemes is a very active line of research. We will consider reduced order models (ROMs) based on Proper Orthogonal Decomposition (POD), which are extensively employed to reduce computational costs in contrast to standard numerical methods. In this work we will prove that, if a standard first-order well-balanced full order model (FOM) of the form

$$W_i^{n+1} = W_i^n - \frac{\Delta t}{\Delta x}\mathcal{L}\left(W_{i-1}^n, W_i^n, W_{i+1}^n\right). \tag{3}$$

is considered, the corresponding reduced order method will be also well-balanced. Moreover, predictions for parameter-dependent systems will be also considered. In particular, the source terms depend on a physical parameter μ. Our interest is to quickly obtain good approximations when considering variations of the parameter μ.