# A Hamilton-Jacobi approach for the evolutionary dynamics of a model with gene transfer. How to characterize monomorphic dynamics for concave fitness functions.

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**Abstract**

During this session we will present recent advances in the study of constrained parabolic Hamilton-Jacobi equations of the form

$$\begin{cases} \partial_t v(t,z) = |\partial_z v(t,z)|^2 + F(t,z), & t \in [0,T], z \in \mathbb{R}^d, \\ \max_{z\in\mathbb{R}} v(t,z) = 0, & t \in [0,T], \\ v(0,z) = v_0(z), & z \in \mathbb{R}^d, \end{cases} \tag{0.1}$$

paying special attention to the case where the function $F$ is not globally concave in time and space due to the appearance of a non-local integral term. We will present the results in the frame of a problem applied to the dynamics of cell populations, modelled by the non-local equation

$$\begin{cases} \varepsilon\partial_t n_\varepsilon(t,z) = \varepsilon^2\partial^2_{zz} n_\varepsilon(t,z) + F(t,z)\cdot n_\varepsilon(t,z), \\ n_\varepsilon(0,z) = n_{\varepsilon,0}(z) > 0, \\ n_\varepsilon(t,z) > 0, \\ \rho_\varepsilon(t) = \displaystyle\int_{\mathbb{R}} n_\varepsilon(t,y)\ \mathrm{dy}. \end{cases} \tag{0.2}$$

The behaviour of the solutions of equation (**??**) as $\varepsilon \to 0$ is related to the behaviour of the solutions of (**??**) via the Hopf-Cole transformation

$$v(t,z) = \lim_{\varepsilon\to 0} \left(\varepsilon \cdot \ln\left(n_\varepsilon(t,z)\right)\right). \tag{0.3}$$

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