Quasi-conservative discontinuous Galerkin schemes for hyperbolic systems in non-conservative variables with subcell finite volume corrections

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Abstract: We present a novel quasi-conservative arbitrary high order accurate ADER discontinuous Galerkin (DG) method allowing to efficiently use a non-conservative form of the considered PDE system, so that it can be modeled directly in the *most physically relevant* set of *variables*. This is useful for *multi-material* flows with moving interfaces and strong contact discontinuities, as well as in presence of very non-linear thermodynamics. Regrettably, the non-conservative formulation introduces a conservation error which would normally lead to a wrong approximation of shock waves. Hence, we start by giving a formal definition of the conservation defect of non-conservative schemes and we analyze it providing a *local quasi-conservation condition*, which allows us to prove a *modified Lax-Wendroff theorem*. Then, to numerically deal with shock waves, we exploit the framework of the so-called *a posteriori* subcell finite volume (FV) limiter, so that, in shock-triggered troubled cells appropriately detected, we can incorporate a local conservation correction. Our corrected FV update entirely removes the local conservation defect allowing to fit in the hypotheses of the proposed modified Lax-Wendroff theorem.

To prove the capabilities of our novel approach, first, we show that we are able to recover the same results given by conservative schemes on classical benchmarks for the single-fluid Euler equations. We then conclude the presentation by showing the improved reliability of our scheme on the *multi-fluid Euler system* on examples like the interaction of a shock with a helium bubble for which we are able to avoid the development of any spurious oscillations, see the figure here below.

E. Gaburro gratefully **acknowledges** the support received from the European Union with the ERC Starting Grant *ALcHyMiA* (grant agreement No. 101114995).