On an Approximate Analytical Solution to a Problem of Radzievskij

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Abstract: In [?] we considered a problem of Celestial Mechanics posed by Radzievskij [?], namely, the problem of motion of two point masses within a homogeneous spherical cloud, supposing that the mutual interaction between these point bodies and between the bodies and the particles of the cloud is described by their Newtonian gravitational attraction. The constant density of the cloud is assumed to be sufficiently small so that the resistance of the medium to the motion of the two point masses can be neglected.

After these simplifying hypotheses, *the problem of relative motion of one of the bodies with respect to the other can be recast as a perturbed Keplerian system, in which the perturbing effects are due to an attractive, conservative central force* (RADZIEVSKIJ, [?]), and the constant density of the cloud can be taken as (proportional to) *a small perturbation parameter*.

In [?] we took different analytical approaches to the formulation and solution of this perturbed Keplerian system. In particular ([?], §7), we established the so–called *Planetary Equations in Gaussian Form* (a system of six first–order ordinary differential equations governing the time variations of the classical Keplerian orbital elements under perturbations), and obtained a first–order analytical solution, in closed form, in terms of the eccentric anomaly E as the independent variable. Such a solution allows us to identify the *first–order secular and periodic variations* undergone by the set of elliptic orbital elements at issue due to the effect of the perturbing force.

In this contributed paper we derive *an alternative set of Gauss Planetary Equations*, on this occasion using *the true anomaly f as the independent variable*, and develop the corresponding first–order solution in terms of f.

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Keywords: Celestial Mechanics, perturbed Keplerian systems, central force, Gauss Planetary Equations, true anomaly, first–order perturbations.

References:

[1] FLORÍA, L.: "On the Motion of Two Point Masses inside a Homogeneous Cloud". *Proceedings of the XXVII CEDYA / XVII CMA* (Zaragoza, Spain, July 18-22, 2022), pp. 133–141. Univ. Zaragoza, 2023.

[2] RADZIEVSKIJ, V. V.: "General Solution of a Case of the Problem of Three Bodies". *Transactions (Doklady) of the USSR Academy of Sciences* (New Series) **91**(6), pp. 1309–1311, 1953 (in Russian).