Influence of a lower order term for the fractional Laplacian BVP in presence of the Hardy potential.

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Abstract: We study existence and regularity of solutions to a problem involving fractional Laplacian and Hardy potential:

$$\begin{cases} (-\Delta)^s u + g(x)|u|^{p-1}u = \lambda \dfrac{u}{|x|^{2s}} + f(x), & \text{in } \Omega, \\ u = 0, & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $0 \in \Omega \subset \mathbb{R}^N$ $(N > 2s)$ is a bounded domain with a smooth boundary, $p > 1$, $\lambda \in \mathbb{R}$, $0 < g \in L^1_{loc}(\Omega)$ and $f \in L_g^{\frac{p+1}{p}}(\Omega)$.

We show the regularizing effect provided by the lower order term: $g(x)|u|^{p-1}u$, as it was done for the Laplacian case, [2]. As a consequence, we improve the results obtained in absence of this lower order term, [1]. More precisely, under the following condition

$$\int_\Omega |x|^{\frac{2(p+1)s}{1-p}} g^{\frac{2}{1-p}} < +\infty,$$

we are able to achieve two noteworthy outcomes concerning our problem:

1. Existence of solutions u for every $\lambda \in \mathbb{R}$.
2. Increasing of the regularity, $u \in H_0^s(\Omega) \cap L_g^{pm}(\Omega)$, when $f \in L_g^m(\Omega)$, for $m \geq \frac{p+1}{p}$.

References:

[1] B. Abdellaoui, M. Medina, I. Peral, A. Primo, *The effect of Hardy potential in some Calderón-Zygmund properties for the fractional Laplacian*, J. Differ. Equ. (2016), **260**, No. 11, pp. 8160-8206.

[2] D. Arcoya, Al. Molino, L. Moreno-Mérida, *Existence and Regularizing Effect of Degenerate Lower Order Terms in Elliptic Equations Beyond the Hardy Constant*, Adv. Nonlinear Stud. (2018), **18**, No. 4, pp. 775–783.