Critical transitions in complex systems: the role of timescales and unstable states

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Abstract:

Many systems in nature exhibit the coexistence of different stable states for a given set of environmental parameters and external forcing. Examples for such behaviour can be found in different fields of science ranging from mechanical or chemical systems to ecosystem and climate dynamics. As a consequence of the coexistence of a multitude of stable states, the final state of the system depends strongly on the initial condition. The set of initial conditions which all converge to the same stable state is called the basin of attraction. In autonomous systems, the boundaries of these basins are made up by the stable manifolds of saddle invariant sets, e.g. saddle fixed points, saddle periodic orbits or chaotic saddles. We analyse systems experiencing a parameter drift during which bifurcations are crossed that lead to the emergence of new attractors including their basins of attraction. Using an ensemble of trajectories, we study the role of the relative size of the non-autonomous basins of attraction and the location of their boundaries in rate-dependent tipping. We demonstrate that the decision whether a trajectory tips or tracks the moving stable state depends crucially on the changes in the non-autonomous basins of attraction, in particular on their boundaries, that also move in state space under a time-dependent variation of a parameter. Our ensemble approach reveals that such bifurcations occurring during the parameter drift might be masked because the relative size of the newly formed non-autonomous basins of attractions goes to zero the slower the rate of the parameter drift. As a consequence, the whole ensemble of initial conditions evolving under parameter drift is not signalling the bifurcation. This phenomenon can be observed for smooth basin boundaries as well as for fractal ones. We show that the relationship between the timescale of the parameter drift and the intrinsic dissipative timescale is responsible for this behaviour.

References:

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