# Parallel Nonlinear Schwarz Domain Decomposition Solvers for Two-Phase Flow in Porous Media

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**Abstract:** In the context of the storage of radioactive waste, the need for advanced parallel nonlinear solvers becomes evident in simulating the neighboring geological environment. These models for two-phase flow equations in porous media are highly nonlinear and are solved over timeframes that could reach millions of years, presenting a significant challenge. Addressing this requirement, innovative nonlinear solvers have been developed and applied, showcasing their effectiveness in handling complex environmental simulations [1]. These solvers stand out due to their use of nonlinear domain decomposition methods, which not only enable efficient parallelization but also significantly improve both nonlinear and linear convergence rates [2, 3]. This is achieved by partitioning the problem into subdomains and solving a smaller subproblem within each. Thanks to an implementation using petsc4py [4], the solvers capabilities were validated through their application to an international benchmark that models the injection of hydrogen into an initially saturated porous medium. This application highlighted the solver's robustness and rapid convergence, essential qualities for the simulation of geological environments. Moreover, the challenge of communication slowdown, which becomes apparent when dealing with a large number of subdomains and negatively affects the linear inner loops, is effectively mitigated. A second-level strategy employing a nonlinear multigrid approach, specifically the Full Approximation Scheme (FAS), addresses this issue. The FAS approach stabilizes the number of inner loops regardless of the number of subdomains, ensuring scalable efficiency suitable for extensive parallel computing applications. Furthermore, the sequence in which operations of the first and second level are applied plays a significant role in enhancing overall performance and convergence. Properly ordering these operations is critical, as it impacts both the effectiveness and efficiency of the method.

**References:**

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