Nonautonomous mathematical modeling of the Allee effect: from bifurcations to critical transitions.

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Abstract: This talk explores mathematical models of critical transitions in nonautonomous scalar differential equations depicting the dynamics of single species populations influenced by the Allee effect. Specifically, we focus on equations where population growth rate exhibits concave derivative with respect to the population size, as seen in several models of this kind. Drawing from the framework of [1], we introduce intrinsic time dependence in both past and future equations. The nonautonomous nature of the model allows for the consideration of phenomena like Earth's rotation or seasonal alternation, potentially impacting species evolution laws. Different types of mechanisms that can produce a critical transition are studied and incorporated into the equation as parametric variations. The critical transition is typically triggered by a saddle-node bifurcation of hyperbolic solutions by varying the parameter. Notably, we examine rate-induced tracking, and illustrate how finite-time Lyapunov exponents can serve as early warning signals for such transitions. This is joint work with I.P. Longo, C. Núñez and R. Obaya.

References:

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