Finite difference schemes for the p-Laplacian

Authors:

- Félix del Teso, Universidad Autónoma de Madrid (felix.delteso@uam.es)
- Erik Lindgren, KTH Royal Institute of Technology (eriklin@kth.se)

Abstract: The aim of this talk is to introduce the topic of finite difference schemes to PDEs related to the p-Laplacian operator

$$\Delta_p u := \nabla(|\nabla u|^{p-2}\nabla u),$$

for $p > 1$. In this collaboration with E. Lindgren ([1]), we first introduce monotone asymptotic expansions of the p-Laplacian. Based on this approximations, we propose a finite difference discretization of the operator ([2]) and apply it to study numerically the associated parabolic problem ([3])

$$\partial_t u - \Delta_p u = f.$$

We show that the explicit scheme is stable, monotone and convergent in the context of viscosity solutions. An important advantage of our approach, is that the CFL-condition makes use of the regularity provided by the scheme to reduce the computational cost. In particular, for Lipschitz data, the CFL-condition is of the same order as for the heat equation and independent of p. Finally, we will discuss extensions obtained in [4] in the context of nonlocal problems.

References:

[1] del Teso, Félix; Lindgren, Erik; A mean value formula for the variational p-Laplacian. *NoDEA Nonlinear Differential Equations Appl.*, 28 (2021), no. 3, Paper No. 27, 33 pp.

[2] del Teso, Félix; Lindgren, Erik; A finite difference method for the variational p-Laplacian *Journal of Scientific Computing*, 90 (2022), Article No. 67, 31pp.

[3] del Teso, Félix; Lindgren, Erik; Finite difference schemes for the parabolic p-Laplace equation *SeMA Journal*, 80(4) (2023), 527–547.

[4] del Teso, Félix; Medina, María; Ochoa, Pablo; Higher-order asymptotic expansions and finite difference schemes for the fractional p-Laplacian *Mathematische Annalen*, (online ready) (2023).