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Helmholtz decomposition based on nonlocal gradients over bounded domains

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Abstract: One of the motivations of nonlocal models is that they may able to provide a framework for more general (or singular) phenomena, since they may not need to use classical derivatives, whereas the Helmholtz decomposition is a relevant result in mathematics and fluid mechanics which states that any (sufficiently smooth) vector field can be written as the sum of curl-free vector field plus a divergence-free one. Thus, the scope of this talk aims to provide a Helmholtz decomposition based on a strongly singular nonlocal gradient. In doing so we continue further with the nonlocal calculus developed for nonlocal gradients defined as

$$D_\delta^s u(x) = c_{n,s} \int_{B(x,\delta)} \frac{u(x)-u(y)}{|x-y|} \frac{x-y}{|x-y|} \frac{w_\delta(x-y)}{|x-y|^{n-1+s}} \, dy, \quad x \in \Omega$$

for $u : \Omega \cup B(0,\delta) \to \mathbb{R}$. This operator keeps a degree of fractional differentiability while providing a framework over bounded domains. Following some previous results regarding nonlocal versions of the Fundamental Theorem of Calculus, Poincaré Sobolev inequalities, Piola identity or compact embedding concerning the operator $D_\delta^s u$, we develop new tools such as nonlocal Divergence and Stokes theorems, three nonlocal Green identities as well as the study of the fundamental solution of the Laplacian given by $\Delta_\delta^s u := \operatorname{div}_\delta^s D_\delta^s u$, for a properly defined nonlocal divergence. Most of them are employed in the path leading to the nonlocal Helmholtz decomposition.

There have been previous results on Helmholtz decompositions concerning two-points nonlocal gradients(without integration), one-point gradient with integrable kernels or the Riesz fractional one (defined over the whole space).

References:

[1] José C. Bellido; Javier Cueto; Mikil Foss; Petronela Radu; *Nonlocal Green theorems and Helmholtz decompositions for truncated fractional gradients* https://arxiv.org/pdf/2311.05465