About non-local reaction-diffusion equations

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Abstract:

Let consider the problem of finding a function $u(t,x)$ such that

$$\frac{\partial u}{\partial t} - a(\int_\Omega u(t,x)dx)\frac{\partial^2 u}{\partial x^2} = g(t,u) \tag{1}$$

In such equation, u could describe the density of a population subject to spreading. The diffusion coefficient a is then supposed to depend on the entire population in the domain rather than on the local density.

It is possible to distinguish two basic cases of (**??**).

If we consider the non-local equation

$$\frac{\partial u}{\partial t} - a(\|u\|^2_{H^1_0})\frac{\partial^2 u}{\partial x^2} = \lambda f(u) \tag{2}$$

with Dirichlet boundary conditions, then it is possible to define a suitable Lypaunov functional. In [**?**] it is shown that regular and strong solutions generate (possibly) multivalued semiflows having a global attractor. In the case where the function f is odd and equation (**??**) generates a continuous semigroup the existence of fixed points of the type given in the Chafee-Infante problem was established in [**?**]. Moreover, if a is non-decreasing, then they coincide with the ones in the Chafee-Infante problem. In this work we extend these results for a more general function f.

If we consider $l \in (L^2(\Omega))'$ a functional acting on u over the whole domain, we obtain results similar to the previous ones about fixed points of the following problem

$$\frac{\partial u}{\partial t} - a(l(u))\frac{\partial^2 u}{\partial x^2} = \lambda f(u)$$

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References:

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