Geometric multigrid for local space-time FEM to 3d Navier–Stokes and dynamic Biot systems

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Abstract: Space-time finite element methods (STFEMs) allow the natural construction of higher order discretizations to systems of flow and multiphysics [1, 3]. They offer the potential to achieve accurate results on computationally feasible grids with a minimum of numerical costs. However, constructing higher order numerical methods maintaining stability and inheriting most of the rich structure of the continuous problem becomes increasingly difficult. Further, to realize higher order schemes that require less CPU time for achieving comparable accuracy, also solvers of optimal complexity, are necessary. Ideally, the approach should also offer robustness with respect to the physical (model) and discretization parameters. We present corresponding solution techniques for the Navier–Stokes equations of incompressible viscous flow and the coupled hyperbolic-parabolic Biot system of dynamic poroelasticity with applications in geophysical and bio-medical engineering. To solve the algebraic systems, geometric multigrid (GMG) preconditioning with local Vanka-type smoother of GMRES iterations is suggested. The performance of the combined STFEM and GMG approach is investigated for challenging three-dimensional test problems [1, 2].

References:

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