Observability and control of parabolic equations on networks with loops

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Abstract: Network theory can be useful for studying complex systems such as those that arise, for example, in physical sciences, engineering, economics and sociology. In this communication, we prove the observability of parabolic equations on networks with loops. By using a novel Carleman inequality, we find that the observability of the entire network can be achieved under certain hypothesis about the position of the observation domain. The main difficulty we tackle, due to the existence of loops, is to avoid entering into a circular fallacy, notably in the construction of the auxiliary function for the Carleman inequality. The difficulty is overcome with a careful treatment of the boundary terms on the junctions. Finally, we use the observability to prove the null controllability of the network and to obtain the Lipschitz stability for an inverse problem consisting on retrieving a stationary potential in the parabolic equation from measurements on the observation domain. This communication is based on the paper [1].

References:

[1] Apraiz, J., & Bárcena-Petisco, J. A. (2023). Observability and control of parabolic equations on networks with loops. Journal of Evolution Equations, 23(2), 37.