A priori estimates for quasilinear elliptic systems

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Abstract: In this talk, based on [?], we will focus on a priori estimates of the type

$$\begin{aligned} u(x) + |Du(x)|^{\alpha_1} &\leq C(1 + dist^{-\alpha_2}(x, \partial\Omega)), \ x \in \Omega \\ v(x) + |Dv(x)|^{\beta_1} &\leq C(1 + dist^{-\beta_2}(x, \partial\Omega)), \ x \in \Omega, \end{aligned} \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N$ is an arbitrary domain, $\alpha_i, \beta_i > 0$ for $i = 1, 2$, for any (u, v) nonnegative solutions of an elliptic system whose prototype is

$$\begin{cases} -\Delta_p u = v^{p_1} - v^{s_1} u^{s_2} |Dv|^{\theta_1} |Du|^{\theta_2} & \text{in } \Omega, \\ -\Delta_q v = u^{q_1} - u^{r_1} v^{r_2} |Du|^{\gamma_1} |Dv|^{\gamma_2} & \text{in } \Omega, \end{cases} \tag{2}$$

with $1 < p, q < N$, $p_1, q_1 > 1$, $s_i, r_i, \theta_i, \gamma_i > 0$ satisfying particular conditions.

Estimates of the type (**??**) are those that Serrin and Zou in [?] call universal a priori estimates, because they are independent of the solutions and do not need any boundary conditions.

The system (**??**) generalizes the celebrated Lane-Emden system, involving quasilinear operators on arbitrary domains of $\mathbb{R}^N$ and a nonlinearity depending on the gradient. Moreover, it is a model in population dynamics used to describe the evolution of the population density of a biological species, under the effect of certain natural mechanism.

The technique used it is based on rescaling arguments combined with a key "doubling" property, which is different from the celebrated blow-up technique due to Gidas and Spruck in [?].

References:

[1] L. Baldelli, R. Filippucci, A priori estimates for elliptic problems via Liouville type theorems, *Discrete Contin. Dyn. Syst. Ser. S, Special Issue on the occasion of the 65th birthday of Patrizia Pucci*, **13**, (2020), 1883–1898.

[2] B. Gidas, J. Spruck, A priori bounds for positive solutions of nonlinear elliptic equations, *Comm. Partial Differential Equations*, **6**, (1981), 883–901.

[3] J. Serrin, H. Zou, Cauchy-Liouville and universal boundedness theorems for quasilinear elliptic equations and inequalities, *Acta Math.*, **189**, (2002), 79–142.