Doubly critical elliptic systems

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Abstract: In this talk we show existence of positive bound and ground states of a Hardy–Sobolev type system of elliptic PDEs coupled by a singular critical Hardy–Sobolev term which reads as

$$\begin{cases} -\Delta u - \lambda_1 \dfrac{u}{|x|^2} - \dfrac{u^{2^*_{s_1}-1}}{|x|^{s_1}} = \nu\alpha h(x) \dfrac{u^{\alpha-1}v^{\beta}}{|x|^{s_3}} & \text{in } \mathbb{R}^N, \\ -\Delta v - \lambda_2 \dfrac{v}{|x|^2} - \dfrac{v^{2^*_{s_2}-1}}{|x|^{s_2}} = \nu\beta h(x) \dfrac{u^{\alpha}v^{\beta-1}}{|x|^{s_3}} & \text{in } \mathbb{R}^N. \end{cases}$$

Here h is a nonnegative function in $\mathbb{R}^N$, $\lambda_1, \lambda_2 > 0$, $\nu > 0$ and $\alpha, \beta > 1$ such that

$$\frac{\alpha}{2^*_{s_1}} + \frac{\beta}{2^*_{s_2}} \leq 1.$$

The main novelty of this work is that the exponents $s_1, s_2, s_3 \in (0, 2)$ are not necessarily equal.

References:

[1] Á. Arroyo, R. López-Soriano, and A. Ortega. *Existence of solutions for a system with general Hardy–Sobolev singular criticalities.* Preprint, 2024. arXiv:2405.20845.