**Rational methods for abstract evolution problems without order reduction**

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**Abstract:** We are concerned with the numerical time integration of abstract, linear, non-homogeneous IVPs of the form

$$\begin{cases} u'(t) &= Au(t) + f(t), \quad 0 \le t \le T, \\ u(0) &= u_0, \end{cases}$$

where $A : D(A) \subset X \to X$ is the infinitesimal generator of a $\mathcal{C}_0$ semigroup of linear and bounded operators in a complex Banach space $X$. This abstract framework covers a wide variety of situations of practical interest, including both parabolic and hyperbolic problems.

It is well known that a Runge-Kutta method of order $p$ applied to one of these IVPs suffers from the so called order reduction phenomenon: it occurs that the method, applied in the context of a solution $u \in \mathcal{C}^{(p+1)}([0,T], X)$, exhibits an order of convergence $\mu \le p$ which is related to the stage order of the method, rather than to $p$ itself.

To overcome this issue, we part from an A-stable rational approximation $r(z)$ to $e^z$ of order $p$ (typically, the stability function of a RK method) and propose a new family of stable methods that recover the order $p$ for every solution $u \in \mathcal{C}^{(p+1)}([0,T], X)$, thus avoiding order reduction. These methods have a similar computational cost than the original RK ones in terms of linear systems required to solve per time step. In addition, only evaluations of the source term and not of its derivatives are required.

Finally, a strategy to extend the methods to semilinear IVPs of the form

$$\begin{cases} u'(t) &= Au(t) + f(t, u(t)), \quad 0 \le t \le T, \\ u(0) &= u_0, \end{cases}$$

with $A$ sectorial, is also presented.

**References:**

[1] C. Arranz-Simon, C. Palencia, *Rational methods for abstract linear, non-homogeneous problems without order reduction.* In preparation.

[2] I. Alonso-Mallo, C. Palencia, *Optimal orders of convergence for Runge–Kutta methods and linear, initial boundary value problems.* Appl. Numer. Math., 44 (2003): 1-19.