# On the regularity of solutions to a slightly subcritical Neumann problem

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**Abstract:** We consider the following Neumann problem

$$\begin{cases} -\Delta u + u = f(x,u), & x \in \Omega\,, \\ \frac{\partial u}{\partial \eta} = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ $(N > 2)$ is a bounded, open domain, with $C^{2,\alpha}$ $(0 < \alpha < 1)$ boundary $\partial\Omega$, $\partial/\partial\eta := \eta(x)\cdot\nabla$ denotes the outer normal dervative on $\partial\Omega$, and the nonlinear reaction term $f : \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ is a *slightly subcritical* Carathéodory function.

Through a De Giorgi-Nash-Moser iteration scheme, it is known that weak solutions to (1) with critical growth are in $L^\infty(\Omega)$.

Our contribution is to provide an explicit $L^\infty(\Omega)$-estimate of weak solutions with slightly subcritical growth, in terms of powers of $H^1(\Omega)$-norms, by combining the elliptic regularity of weak solutions with Gagliardo–Nirenberg interpolation inequality.

**Keywords:** De Giorgi-Nash-Moser estimate; Sobolev embedding; Hölder inequality.

**References:**

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