Ariketa 1	Ariketa 2	Ariketa 3	Ariketa 4	Ariketa 5	Ariketa 6	1. Zatia

Azterketaren iraupena: 3 ordu

IZEN-ABIZENAK:

TALDEA:

1.- Kalkulatu $\lim_{n\to\infty} \sqrt[n]{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}$

(0.75 puntu)

$$\lim_{n\to\infty} \sqrt[n]{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}} \overset{(Z-E)}{=} \lim_{n\to\infty} \frac{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n-1}} \overset{(Stolz)}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n-1}} = 1$$

Stolz erabil daiteke $\left\{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n-1}\right\}$ hertsiki gorakorra eta dibergentea baita.

2.- Aztertu $\sum_{n=1}^{\infty} \frac{1}{a^n+2}$ **seriearen izaera** $\forall a \geq 0$**.**

(0.75 puntu)

$\sum_{n=1}^{\infty} a_n$ non $a_n = \frac{1}{a^n+2} \geq 0 \quad \forall n$

$$\lim_{n\to\infty} a^n = \begin{cases} \infty & \forall a > 1 \\ 1 & \text{baldin } a = 1 \\ 0 & \forall a < 1 \end{cases} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{a^n+2} \begin{cases} 0 & \forall a > 1 \\ \frac{1}{3} & \text{baldin } a = 1 \\ \frac{1}{2} & \forall a < 1 \end{cases}$$

- $\forall a \leq 1 \quad \lim_{n\to\infty} a_n \neq 0 \quad \Rightarrow \quad \sum_{n=1}^{\infty} a_n$ dibergentea da.
- $\forall a > 1 \quad a_n = \frac{1}{a^n+2} \sim \frac{1}{a^n} = \left(\frac{1}{a}\right)^n \quad \Rightarrow \quad \sum_{n=1}^{\infty} \left(\frac{1}{a}\right)^n$ serie geometrikoa da, arrazoia $r = \frac{1}{a} < 1$ izanik, beraz, $\sum_{n=1}^{\infty} \left(\frac{1}{a}\right)^n$ konbergentea da $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{a^n+2}$ konbergentea da.

3.- $\displaystyle\sum_{n=1}^{\infty}\frac{2^n\cdot x^n}{n}$ **berretura-seriea emanik:**

a) Aurkitu bere konbergentzi arloa.

b) Kalkulatu bere batura konbergentea den eremuan.

(1.5 puntu)

a) Konbergentzi arloa lorrtzeko, D´Alembert aplikatuko diogu balio absolutuen serieari:

$$\lim_{n\to\infty}\frac{\dfrac{2^{n+1}\cdot|x|^{n+1}}{n+1}}{\dfrac{2^n\cdot|x|^n}{n}}=2|x|<1\quad\Leftrightarrow\quad|x|<\frac{1}{2}\quad\Leftrightarrow\quad-\frac{1}{2}<x<\frac{1}{2}$$

Baldin $x=\frac{1}{2}\quad\Rightarrow\quad\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ dibergentea da.

Baldin $x=-\frac{1}{2}\quad\Rightarrow\quad\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ baldintzaz konbergentea da.

Orduan, $\displaystyle\sum_{n=1}^{\infty}\frac{2^n\cdot x^n}{n}$ konbergentea da $\forall x\in\left[-\frac{1}{2},\frac{1}{2}\right)$.

b) Beraz, $\exists S(x)=\displaystyle\sum_{n=1}^{\infty}\frac{2^n\cdot x^n}{n}\quad\forall x\in\left[-\frac{1}{2},\frac{1}{2}\right)$, eta deribagarria da $\forall x\in\left(-\frac{1}{2},\frac{1}{2}\right)$. Hau da:

$$\exists S'(x)=\sum_{n=1}^{\infty}2^n\cdot x^{n-1}\overset{(*)}{=}\frac{2}{1-2x}\quad\forall x\in\left(-\frac{1}{2},\frac{1}{2}\right)$$

Eta integratuz:

$$\left.\begin{array}{l}
S(x)-\underbrace{S(0)}_{=0}=\displaystyle\int_0^x\frac{2}{1-2x}dt=-\mathrm{L}(1-2x)=\sum_{n=1}^{\infty}\frac{2^n\cdot x^n}{n}\quad\forall x\in\left(-\frac{1}{2},\frac{1}{2}\right)\\[2ex]
x=-\dfrac{1}{2}\text{ puntuan }\displaystyle\sum_{n=1}^{\infty}\frac{2^n\cdot x^n}{n}\text{ konbergentea da berez, } S \text{ funtzio jarraitua da.}\\[2ex]
x=-\dfrac{1}{2}\text{ puntuan }-\mathrm{L}(1-2x)\text{ funtzio jarraitua da.}
\end{array}\right\}$$

Beraz, $S(x)=\displaystyle\sum_{n=1}^{\infty}\frac{2^n\cdot x^n}{n}=-\mathrm{L}(1-2x)\quad\forall x\in\left[-\frac{1}{2},\frac{1}{2}\right)$

(*) Serie geometrikoa da, bere arrazoia $r=2x\quad\Rightarrow\quad|r|<1\quad\forall x\in\left(-\frac{1}{2},\frac{1}{2}\right)$

4.- Kalkulatu $\lim_{x\to\infty}\left(x^{1/x^3}-1\right)\cdot \mathrm{L}x$

(0.75 puntu)

$$\lim_{x\to\infty} x^{1/x^3} = \infty^0 = A \quad \Leftrightarrow \quad \mathrm{L}A = \lim_{x\to\infty}\frac{\mathrm{L}x}{x^3} = 0 \quad \Leftrightarrow \quad A = e^0 = 1 \qquad (1)$$

$$\lim_{x\to\infty}\left(x^{1/x^3}-1\right)\cdot \mathrm{L}x \stackrel{(1)}{=} \lim_{x\to\infty}\mathrm{L}\left(x^{1/x^3}\right)\cdot \mathrm{L}x = \lim_{x\to\infty}\frac{1}{x^3}\mathrm{L}x\cdot \mathrm{L}x = \lim_{x\to\infty}\frac{\left(\mathrm{L}x\right)^2}{x^3} = 0$$

5.- Aurkitu analitiko eta grafikoki hurrengo funtzioaren definizio-eremua:

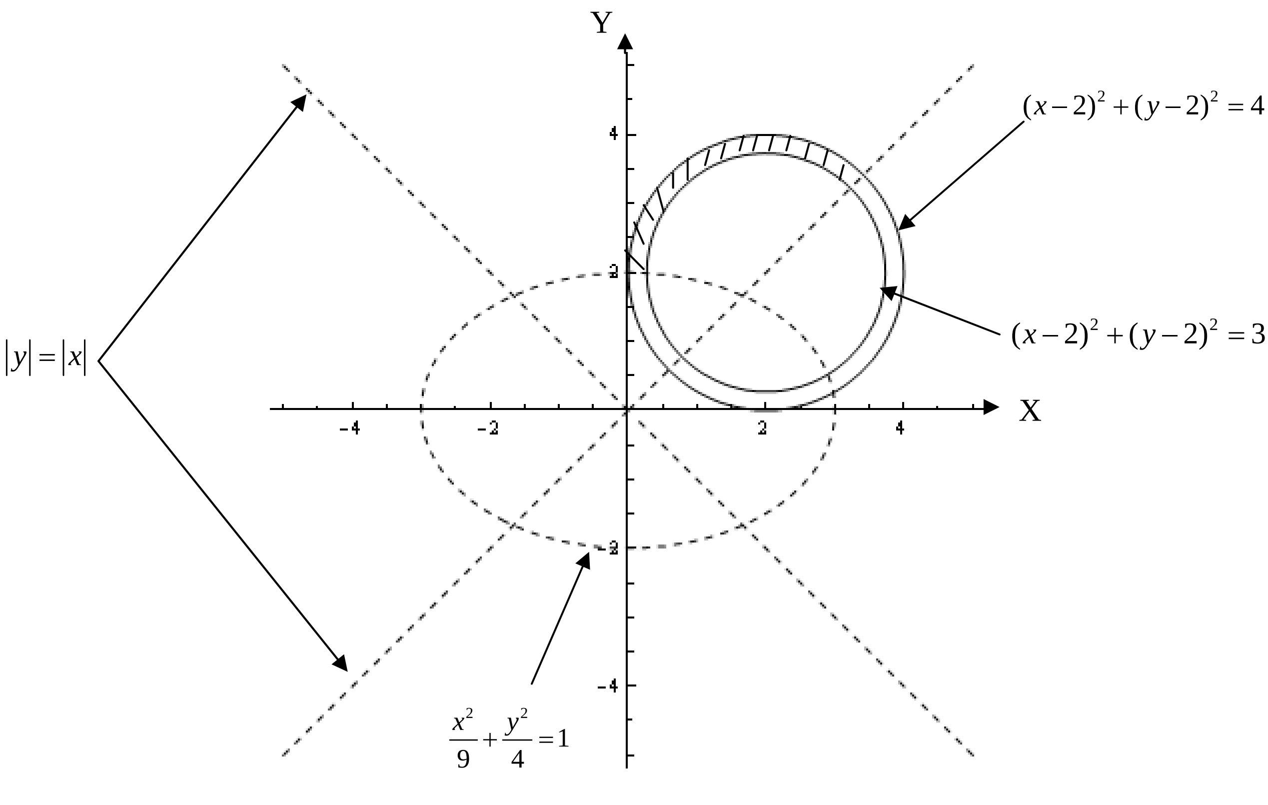
$$f(x,y)=\arcsin\left(\sqrt{(x-2)^2+(y-2)^2-3}\right)+\frac{\mathrm{L}\left(|y|-|x|\right)}{\sqrt{4x^2+9y^2-36}}$$

(1.5 puntu)

$$D=\left\{\begin{array}{l}(x,y)\in\mathbb{R}^2 \,/\, -1\le\sqrt{(x-2)^2+(y-2)^2-3}\le 1,\\ \qquad (x-2)^2+(y-2)^2-3\ge 0,\\ \qquad |y|-|x|>0,\\ \qquad 4x^2+9y^2-36>0\end{array}\right\}$$

- $-1\le\sqrt{(x-2)^2+(y-2)^2-3}\le 1 \iff \sqrt{(x-2)^2+(y-2)^2-3}\le 1 \iff$

  $\iff (x-2)^2+(y-2)^2-3\le 1 \iff (x-2)^2+(y-2)^2\le 4$
- $(x-2)^2+(y-2)^2-3\ge 0 \iff (x-2)^2+(y-2)^2\ge 3$
- $|y|-|x|>0 \iff |y|>|x|$
- $4x^2+9y^2-36>0 \iff 4x^2+9y^2>36 \iff \dfrac{x^2}{9}+\dfrac{y^2}{4}>1$



6.- Aztertu $f(x,y)=\begin{cases}\dfrac{y^3\cdot e^{x+y}}{x^2+y^2} & \forall (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$ **funtzioaren jarraitutasuna eta diferentziagarritasuna** $(0,0)$ **puntuan.**

(1.75 puntu)

f jarraitua da $(0,0)$ puntuan $\Leftrightarrow \lim\limits_{(x,y)\to(0,0)} f(x,y)=f(0,0)\in\mathbb{R}$

$$\lim_{(x,y)\to(0,0)} f(x,y)=\lim_{(x,y)\to(0,0)}\frac{y^3\cdot \overbrace{e^{x+y}}^{1}}{x^2+y^2}=\lim_{(x,y)\to(0,0)}\frac{y^3}{x^2+y^2}=\lim_{\substack{\rho\to0^+\\ \forall\theta\in[0,2\pi)}}\frac{\rho^3\cdot\sin^3\theta}{\rho^2}=0=f(0,0)$$

Beraz, f jarraitua da $(0,0)$ puntuan.

f –ren diferentziagarritasuna aztertzeko baldintza beharrezkoa eta nahikoa erabiliko dugu, eta, horretarako, lehendabizi, deribatu partzialak kalkulatu behar dira:

$$f'_x(0,0)=\lim_{h\to0}\frac{f(h,0)-f(0,0)}{h}=\lim_{h\to0}\frac{\frac{0}{h^2}}{h}=0$$

$$f'_y(0,0)=\lim_{k\to0}\frac{f(0,k)-f(0,0)}{k}=\lim_{k\to0}\frac{\frac{k^3\cdot e^k}{k^2}}{k}=\lim_{k\to0}e^k=1$$

Orain B.B.N:

$$\lim_{(h,k)\to(0,0)}\frac{\left|f(h,k)-f(0,0)-h\cdot f'_x(0,0)-k\cdot f'_y(0,0)\right|}{\sqrt{h^2+k^2}}=\lim_{(h,k)\to(0,0)}\frac{\left|\frac{k^3\cdot e^{h+k}}{h^2+k^2}-k\right|}{\sqrt{h^2+k^2}}=$$

$$=\lim_{\substack{\rho\to0^+\\ \forall\theta\in[0,2\pi)}}\frac{\left|\frac{\rho^3\cdot\sin^3\theta\cdot e^{\rho(\cos\theta+\sin\theta)}}{\rho^2}-\rho\cdot\sin\theta\right|}{\rho}=\lim_{\substack{\rho\to0^+\\ \forall\theta\in[0,2\pi)}}\left|\sin^3\theta\cdot e^{\rho(\cos\theta+\sin\theta)}-\sin\theta\right|=$$

$$=\left|\sin^3\theta-\sin\theta\right|\neq0$$

Beraz, f ez da diferentziagarria $(0,0)$ puntuan.

Ariketa 7	Ariketa 8	Ariketa 9	Ariketa 10	Ariketa 11	2. Zatia

Azterketaren iraupena: 3 ordu

IZEN-ABIZENAK:

TALDEA:

7.- Kalkulatu $f(x,y)=x\cdot y^2+g\left(y\cdot e^{x^2\cdot y}\right)$ **funtzio diferentziagarriaren deribatu direkzional maximoa** $P(x,y)=(1,1)$ **puntuan, jakinda** $g(1)=g'(1)=1$ **eta** $g(e)=g'(e)=\dfrac{-1}{2e}$**.**

(1.75 puntu)

f-ren deribatu direkzional maximoa $P(x,y)=(1,1)$ puntuan $\left|\overrightarrow{\nabla f}(1,1)\right|$ da.

$$f_x'(x,y)=y^2+2xy^2\cdot e^{x^2\cdot y}\cdot g'\left(y\cdot e^{x^2\cdot y}\right) \Rightarrow f_x'(1,1)=1+2e\cdot g'(e)=1+2e\left(\frac{-1}{2e}\right)=0$$

$$f_y'(x,y)=2xy+\left(e^{x^2\cdot y}+yx^2\cdot e^{x^2\cdot y}\right)\cdot g'\left(y\cdot e^{x^2\cdot y}\right)=2xy+\left(1+yx^2\right)\cdot e^{x^2\cdot y}\cdot g'\left(y\cdot e^{x^2\cdot y}\right) \Rightarrow$$

$$\Rightarrow f_y'(1,1)=2+(1+1)\cdot e\cdot g'(e)=2+2e\left(\frac{-1}{2e}\right)=1$$

Orduan, $\left|\overrightarrow{\nabla f}(1,1)\right|=\sqrt{\left(f_x'(1,1)\right)^2+\left(f_1'(1,1)\right)^2}=1$

8.- $\begin{cases} x^2+2y^3-z+1=0 \\ 2x+3y+z=0 \end{cases}$ **ekuazio-sistema emanik,**

a) Aurkitu $a,b\in\mathbb{R}$ parametroen balioak, sistema horrek x aldagaiko y eta z funtzio inplizituak ($y=y(x)$, $z=z(x)$) defini ditzan $P(x,y,z)=(a,0,b)$ puntuan.

b) Izan bedi $h(x)=x^2+y(x)-z(x)$. Egiaztatu funtzio honek mutur erlatiboa duela $x=-1$ puntuan, eta sailkatu.

(2.25 puntu)

a) Funtzio inplizituaren teorema aplikatuko diogu $\begin{cases} F(x,y,z)=x^2+2y^3-z+1=0 \\ G(x,y,z)=2x+3y+z=0 \end{cases}$ sistemari:

i. $\begin{cases} F(P)=a^2-b+1=0 \\ G(P)=2a+b=0 \end{cases} \Leftrightarrow \ a^2+2a+1=(a+1)^2=0 \ \Leftrightarrow \ \boxed{a=-1 \ \Leftrightarrow \ b=2}$

ii. $\begin{cases} F'_x=2x \quad F'_y=6y^2 \quad F'_z=-1 \\ G'_x=2 \quad G'_y=3 \quad G'_z=1 \end{cases}$ existitzen eta jarraituak dira P puntuaren ingurunean.

iii. $\left|\dfrac{D(F,G)}{D(y,z)}\right|_P = \begin{vmatrix} F'_y & F'_z \\ G'_y & G'_z \end{vmatrix}_P = \begin{vmatrix} 6y^2 & -1 \\ 3 & 1 \end{vmatrix}_P = \begin{vmatrix} 0 & -1 \\ 3 & 1 \end{vmatrix} = 3 \neq 0$

Beraz, $P(x,y,z)=(a,0,b)=(-1,0,2)$ puntuaren ingurunean, $\exists! \begin{cases} y=y(x) \\ z=z(x) \end{cases}$, non y eta z funtzio diferentziagarriak diren eta $\begin{cases} y(-1)=0 \\ z(-1)=2 \end{cases}$.

b) $h(x)=x^2+y(x)-z(x)$ funtzioak $x=-1$ puntuan mutur erlatiboa izateko, lehenengo puntu kritikoa dela frogatu behar dugu, hau da $h'(-1)=0$.

$$h'(x)=2x+y'(x)-z'(x)$$

$\begin{cases} F(x,y(x),z(x))=0 \\ G(x,y(x),z(x))=0 \end{cases}$ sisteman x-rekiko deribatuz:

$$\begin{cases} 2x+6y^2\cdot y'-z'=0 \\ 2+3y'+z'=0 \end{cases} \overset{P \text{ puntuan}}{\Rightarrow} \begin{cases} -2-z'(-1)=0 \\ 2+3y'(-1)+z'(-1)=0 \end{cases} \Leftrightarrow \begin{cases} \boxed{z'(-1)=-2} \\ 2+3y'(-1)-2=0 \end{cases}$$

$\Leftrightarrow \ \boxed{y'(-1)=0}$

Beraz, $h'(-1)=-2+y'(-1)-z'(-1)=-2+0+2=0$.

Behin frogatu $x=-1$ puntu kritikoa dela, orain sailkatuko dugu. Horretarako $h''(-1)$ lortu behar da.

$$\begin{cases} 2x + 6y^2 \cdot y' - z' = 0 \\ 2 + 3y' + z' = 0 \end{cases}$$ sisteman berriro *x*-rekiko deribatuz:

$$\begin{cases} 2 + 12y \cdot \left(y'\right)^2 + 6y^2 \cdot y'' - z'' = 0 \\ 3y'' + z'' = 0 \end{cases} \overset{P \text{ puntuan}}{\Rightarrow} \begin{cases} 2 - z''(-1) = 0 \quad \Leftrightarrow \quad \boxed{z''(-1) = 2} \\ 3y''(-1) + z''(-1) = 0 \quad \Leftrightarrow \quad \boxed{y''(-1) = -\dfrac{2}{3}} \end{cases}$$

$$h''(x) = 2 + y''(x) - z''(x) \quad \Rightarrow \quad h''(-1) = 2 + y''(-1) - z''(-1) = 2 - \frac{2}{3} - 2 = -\frac{2}{3} < 0$$

Beraz, *h* funtzioak maximo erlatiboa du $x = -1$ puntuan.

9.- Kalkulatu $\int_{-1}^{1} \frac{1}{x^2} dx$ integrala.

(Puntu 1)

$\int_{-1}^{1} f(x)dx$ non $f(x) = \frac{1}{x^2} \in \mathbb{R} \quad \forall x \in [-1,1] - \{0\}$

$\lim_{x \to 0} f(x) = \infty \quad \Rightarrow \quad x = 0$ puntu singularra da.

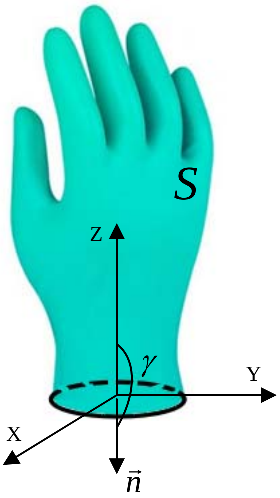
Beraz, integral inpropioa dugu:

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx = I_1 + I_2$$

$I_1 = \int_{-1}^{0} \frac{dx}{x^2} = -\frac{1}{x}\Big|_{-1}^{0} = \infty \quad \Rightarrow \quad I_1$ dibergentea da. Beraz, I dibergentea da.

10.- Kalkulatu $\vec{F}(x,y,z) = 2xy\cdot\vec{i} - y^2\cdot\vec{j} + \vec{k}$ bektorearen fluxua marrazkian erakusten den *S* gainazal irekia eta leunean zehar, $z=0$ planoan $x^2+y^2=1$ zirkunferentziak mugaturikoa.

(1.75 puntu)



Izan bedi $S_2 \equiv z=0 \quad \forall(x,y)\in R_{xy} \equiv x^2+y^2 \le 1$

Eta izan bedi $S' = S \cup S_2$ gainazal zatika leuna eta itxia.

$\vec{F}$ bektorearen fluxua $S' = S \cup S_2$ gainazalean zehar honako hau da:

$$\iint_{S'} \vec{F}\cdot d\vec{S} = \iint_{S} \vec{F}\cdot d\vec{S} + \iint_{S_2} \vec{F}\cdot d\vec{S} \overset{(Gauss)}{=} \iiint_V \underbrace{div\left(\vec{F}\right)}_{=0} dxdydz = 0 \quad \Leftrightarrow \quad \iint_{S} \vec{F}\cdot d\vec{S} = -\iint_{S_2} \vec{F}\cdot d\vec{S}$$

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \iint_{S_2} \left(2xy\cdot dydz - y^2\cdot dzdx + dxdy\right) \overset{(*)}{=} -\iint_{R_{xy}} dxdy = -\text{Azalera}\left(R_{xy}\right) = -\pi$$

Beraz, $\iint_{S} \vec{F}\cdot d\vec{S} = \pi$

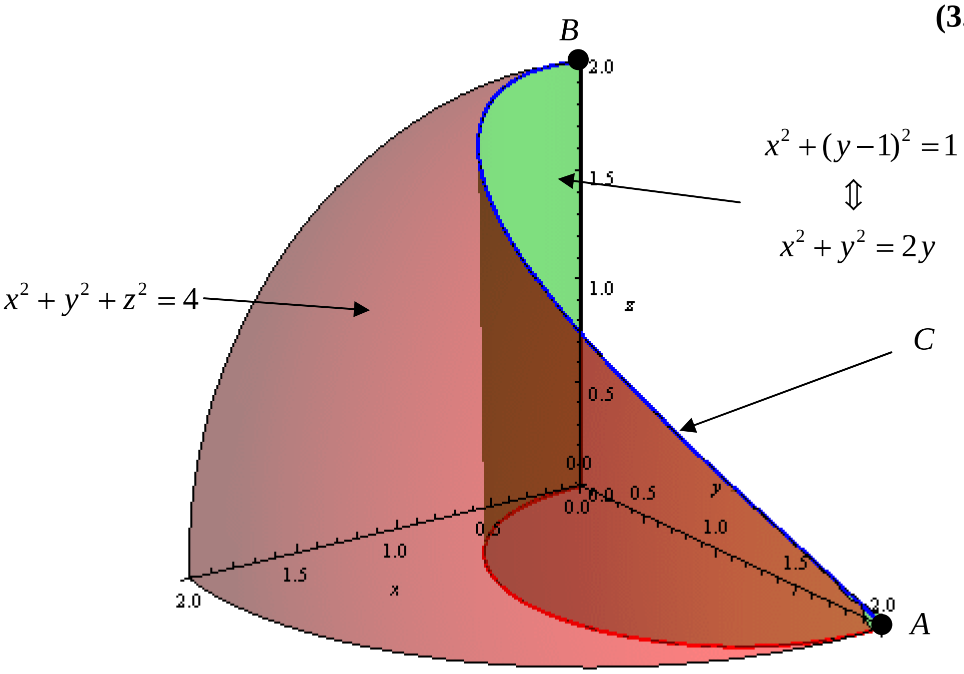
(*) $S_2 \equiv z=0 \quad \forall(x,y)\in R_{xy} \equiv x^2+y^2 \le 1 \quad \Rightarrow \quad dz=0$

Eta $\gamma > \dfrac{\pi}{2}$

11.- **a) Kalkulatu** $V \equiv \begin{cases} x^2+y^2+z^2 \le 4 \\ x^2+(y-1)^2 \ge 1 \\ x \ge 0 \quad y \ge 0 \quad z \ge 0 \end{cases}$ **solidoaren bolumena.**

b) Kalkulatu $\vec{F}(x,y,z) = 2xy \cdot \vec{i} - (y+1)^2 \cdot \vec{j} + 3z \cdot \vec{k}$ **bektorearen lerro-integrala** $C \equiv \begin{cases} x^2+y^2+z^2 = 4 \\ x^2+(y-1)^2 = 1 \end{cases}$ **kurban zehar, lehenengo oktantean,** $A(0,2,0)$ **puntutik** $B(0,0,2)$ **puntura.**

(3.25 puntu)



a) Zilindrikoetan:

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases} \quad |J| = \rho \quad \Rightarrow \quad V \equiv \begin{cases} \rho^2 + z^2 \le 4 \\ \rho^2 \ge 2\rho\sin\theta \\ \cos\theta \ge 0 \quad \sin\theta \ge 0 \quad z \ge 0 \end{cases} \quad \Leftrightarrow \quad V \equiv \begin{cases} 0 \le \theta \le \pi/2 \\ 2\sin\theta \le \rho \le 2 \\ 0 \le z \le \sqrt{4-\rho^2} \end{cases}$$

$$\text{Bolumena}(V) = \iiint_V dxdydz = \int_0^{\pi/2}\int_{2\sin\theta}^{2} \rho\sqrt{4-\rho^2}\,d\rho\,d\theta = \int_0^{\pi/2} \frac{\left(4-\rho^2\right)^{3/2}}{3/2}\cdot\left(\frac{-1}{2}\right)\Bigg|_{2\sin\theta}^{2} d\theta =$$

$$= \int_0^{\pi/2} \frac{\left(4-4\sin^2\theta\right)^{3/2}}{3}\,d\theta = \frac{8}{3}\int_0^{\pi/2}\cos^3\theta\,d\theta = \frac{8}{3}\int_0^{\pi/2}\cos\theta\left(1-\sin^2\theta\right)d\theta = \frac{8}{3}\left[\sin\theta - \frac{\sin^3\theta}{3}\right]_0^{\pi/2} =$$

$$= \frac{8}{3}\left(1-\frac{1}{3}\right) = \frac{16}{9}$$

Integrazio ordena aldatuz:

$$C \equiv \begin{cases} x^2+y^2+z^2=4 \\ x^2+(y-1)^2=1 \end{cases} \equiv \begin{cases} \rho^2+z^2=4 \\ \rho=2\sin\theta \end{cases} \equiv \begin{cases} 4\sin^2\theta+z^2=4 \\ \rho=2\sin\theta \end{cases} \equiv \begin{cases} z=\sqrt{4-4\sin^2\theta} \\ \rho=2\sin\theta \end{cases} \equiv \begin{cases} z=2\cos\theta \\ \rho=2\sin\theta \end{cases}$$

$$\Rightarrow \quad V \equiv \begin{cases} 0\le\theta\le\pi/2 \\ 0\le z\le 2\cos\theta \\ 2\sin\theta\le\rho\le\sqrt{4-z^2} \end{cases} \quad \Rightarrow \text{Bolumena}(V)=\iiint_V dxdydz = \int_0^{\pi/2}\int_0^{2\cos\theta}\int_{2\sin\theta}^{\sqrt{4-z^2}} \rho\, d\rho\, dz\, d\theta$$

b) $$C \equiv \begin{cases} x^2+y^2+z^2=4 \\ x^2+(y-1)^2=1 \end{cases} \equiv \begin{cases} x^2+y^2+z^2=4 \\ x^2+y^2=2y \end{cases} \equiv \begin{cases} x^2+(y-1)^2=1 \\ z=\sqrt{4-2y} \end{cases} \equiv \begin{cases} x=\cos t \\ y=1+\sin t \\ z=\sqrt{2-2\sin t} \end{cases}$$

non $t\,\frac{\pi}{2}\to-\frac{\pi}{2}$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C \left(2xydx-(y+1)^2dy+3zdz\right) = \int_C 2xydx - \int_C (y+1)^2dy + \int_C 3zdz$$

- $$\int_C 2xydx = \int_{\pi/2}^{-\pi/2} -2\cos t\cdot\sin t\cdot(1+\sin t)dt = \cos^2 t - \frac{2\sin^3 t}{3}\Bigg|_{\pi/2}^{-\pi/2} = -\left(-\frac{2}{3}-\frac{2}{3}\right)=\frac{4}{3}$$
- $$\int_C (y+1)^2dy = \int_2^0 (y+1)^2dy = \frac{(y+1)^3}{3}\Bigg|_2^0 = \frac{1}{3}-9=-\frac{26}{3}$$
- $$\int_C 3zdz = \int_0^2 3zdz = \frac{3z^2}{2}\Bigg|_0^2 = 6$$

Beraz, $\int_C \vec{F}\cdot d\vec{r} = \frac{4}{3}+\frac{26}{3}+6=16$