Ariketa 1	Ariketa 2	Ariketa 3	1. zatia

Azterketaren iraupena: 3 ordu

IZEN-ABIZENAK:

TALDEA:

1.- Izan bedi $w(x,y)=e^{g(y^2)}+f\left(y^2+g(xy)\right)$**,** f **eta** g **funtzio diferentziagarriak direlarik. Jakinda** $g(1)=0$**,** $g'(1)=1$ **eta** $f'(1)=2$ **direla, kalkulatu hurrengo adierazpenaren balioa:**

$$E\equiv w'_y(1,1)-w'_x(1,1)$$

(Puntu 1)

$$w(x,y)=e^{g(y^2)}+f\left(y^2+g(xy)\right)\;\Rightarrow\;\begin{cases}w'_x=y\cdot g'(xy)\cdot f'\left(y^2+g(xy)\right)\\ w'_y=2y\cdot g'(y^2)\cdot e^{g(y^2)}+(2y+x\cdot g'(xy))\cdot f'\left(y^2+g(xy)\right)\end{cases}$$

$$\Rightarrow\;\begin{cases}w'_x(1,1)=g'(1)\cdot f'\left(1+g(1)\right)=g'(1)\cdot f'(1)=2\\ w'_y(1,1)=2g'(1)\cdot e^{g(1)}+(2+g'(1))\cdot f'\left(1+g(1)\right)=8\end{cases}$$

Orduan: $E\equiv w'_y(1,1)-w'_x(1,1)=8-2=6$

2.- $\begin{cases} F(x,y,z,u) = xu + yz + 1 = 0 \\ G(x,y,z,u) = x^2 - y^2 + z - u^2 = 0 \end{cases}$ **sistema emanik:**

a) Estudiatu $z = z(x,y)$ **eta** $u = u(x,y)$ **funtzioak definitzen ote dituen** $P(x,y,z,u) = \left(\frac{1}{2}, -\frac{1}{2}, 1, -1\right)$ **puntuaren ingurune batean.**

b) Kalkulatu z'_x **eta** u'_x $Q(x,y) = \left(\frac{1}{2}, -\frac{1}{2}\right)$ **puntuan**

(2 puntu)

a) Funtzio inplizituaren teorema aplikatuko diogu emandako ekuazio-sistemari:

i. $\begin{cases} F(P) = -\frac{1}{2} - \frac{1}{2} + 1 = 0 \\ G(P) = \frac{1}{4} - \frac{1}{4} + 1 - 1 = 0 \end{cases}$

ii. $\begin{cases} F'_x = u & F'_y = z & F'_z = y & F'_u = x \\ G'_x = 2x & G'_y = -2y & G'_z = 1 & G'_u = -2u \end{cases}$ existitzen eta jarraituak dira P puntuaren ingurune batean.

iii. $\left|\frac{D(F,G)}{D(z,u)}\right|_P = \begin{vmatrix} y & x \\ 1 & -2u \end{vmatrix}_P = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ 1 & 2 \end{vmatrix} = -\frac{3}{2} \neq 0$

Beraz, $Q(x,y) = \left(\frac{1}{2}, -\frac{1}{2}\right)$ puntuaren ingurune batean $\exists! \begin{cases} z = z(x,y) \\ u = u(x,y) \end{cases}$ diferentziagarria, non $\begin{cases} z(Q) = 1 \\ u(Q) = -1 \end{cases}$.

b) Emandako sisteman x-rekiko deribatuz:

$$\begin{cases} u + x \cdot u'_x + y \cdot z'_x = 0 \\ 2x + z'_x - 2u \cdot u'_x = 0 \end{cases} \overset{P \text{ puntuan}}{\Rightarrow} \begin{cases} -1 + \frac{1}{2} \cdot u'_x(Q) - \frac{1}{2} \cdot z'_x(Q) = 0 \quad (\times 2) \\ 1 + z'_x(Q) + 2 \cdot u'_x(Q) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -2 + u'_x(Q) - z'_x(Q) = 0 \\ 1 + z'_x(Q) + 2 \cdot u'_x(Q) = 0 \end{cases} \overset{\text{Batuz}}{\Rightarrow} -1 + 3 \cdot u'_x(Q) = 0 \iff u'_x(Q) = \frac{1}{3} \Rightarrow z'_x(Q) = -\frac{5}{3}$$

3.- Garesti samarra den material isolatzailea erabiliz, *R* erradioko eta *h* altuerako ontzi zilindriko itxia eraiki nahi da, $2\pi\, m^3$-ko kapazitatekoa. Aurkitu *R* eta *h*, ontzi bakoitza eraikitzeko beharrezkoa den materialaren kantitatea minimoa izan dadin.

(2 puntu)

Ontzi bakoitzerako, azalera minimoa izan behar dugu lortu nahi dugun kapazitatea mantenduz. Beraz, *f* funtzioaren minimo erlatibo baldintzatua bilatzen ari gara non:

Zilindro itxiaren azalera $f(R,h) = 2\pi Rh + 2\pi R^2$ funtzioak ematen du.

Eta baldintza $\varphi(R,h) = \pi R^2 h - 2\pi = 0 \quad \Leftrightarrow \quad R^2 h - 2 = 0$ ekuazioak adierazten du.

Bi eratan egin daiteke:

- Lagrange-ren biderkatzaileen metodoa erabiliz:

$$w(R,h) = 2\pi Rh + 2\pi R^2 + \lambda(R^2 h - 2)$$

Puntu kritikoak:

$$\left.\begin{cases} w'_R = 2\pi h + 4\pi R + 2\lambda Rh = 0 \\ w'_h = 2\pi R + \lambda R^2 = 0 \quad \Leftrightarrow \quad R(2\pi + \lambda R) = 0 \quad \overset{R>0}{\Rightarrow} \quad \lambda = -\dfrac{2\pi}{R} \\ R^2 h - 2 = 0 \end{cases}\right\} \Rightarrow \quad \pi h + 2\pi R - \frac{2\pi}{R} Rh = 0$$

$$\Rightarrow \quad 2\pi R - \pi h = 0 \quad \Rightarrow \quad \left.\begin{matrix} h = 2R \\ R^2 h - 2 = 0 \end{matrix}\right\} \Rightarrow \quad 2R^3 - 2 = 0 \quad \Leftrightarrow \quad R = 1 \quad \Rightarrow \quad h = 2$$

Orain, $P(R,h) = (1,2)$ puntu kritikoa sailkatu behar dugu:

$$\left.\begin{matrix} \begin{cases} w''_{R^2} = 4\pi + 2\lambda h \quad \Rightarrow \quad w''_{R^2}(P) = -4\pi \\ w''_{h^2} = 0 \qquad\qquad\qquad\qquad \Rightarrow \quad d^2 w(P) = -4\pi (dR)^2 - 4\pi dRdh \\ w''_{Rh} = 2\pi + 2\lambda R = 0 \quad \Rightarrow \quad w''_{Rh}(P) = -2\pi \end{cases} \\ \text{Baina} \quad R^2 h - 2 = 0 \quad \Rightarrow \quad 2RhdR + R^2 dh = 0 \quad \overset{P}{\Rightarrow} \quad 4dR + dh = 0 \quad \Rightarrow \quad dh = -4dR \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \quad d^2 w(P) = -4\pi (dR)^2 + 16\pi (dR)^2 = 12\pi (dR)^2 > 0$$

Beraz, $P(R,h) = (1,2)$ minimoa da.

- $R^2 h - 2 = 0$ ekuazioatik *h* *R*-ren mende bakanduz eta *f* funtzioaren adierazpenean ordezkatuz:

$$R^2 h - 2 = 0 \quad \Leftrightarrow \quad h = \frac{2}{R^2} \quad \Rightarrow \quad f(R,h) = 2\pi R \frac{2}{R^2} + 2\pi R^2 = 2\pi \left(\frac{2}{R} + R^2 \right) = F(R)$$

Puntu kritikoak:

$$F'(R) = 2\pi \left(-\frac{2}{R^2} + 2R \right) = \frac{4\pi}{R^2} \left(-1 + R^3 \right) = 0 \quad \Leftrightarrow \quad R^3 = 1 \quad \Leftrightarrow \quad R = 1 \quad \Rightarrow \quad h = 2$$

Orain, $P(R,h) = (1,2)$ puntu kritikoa sailkatu behar dugu:

$$F''(R) = 4\pi \left(\frac{2}{R^3} + 1 \right) \quad \Rightarrow \quad F''(1) = 12\pi > 0 \quad \Rightarrow \text{ minimoa da.}$$

Ariketa 4	Ariketa 5	Ariketa 6	2. zatia

Azterketaren iraupena: 3 ordu

IZEN-ABIZENAK:

TALDEA:

4.- a) Aurkitu $\lambda \in \mathbb{R} - \{0\}$ balioak, $F(\lambda) = \int_1^\infty e^{\lambda x} dx$ integral inpropioa konbergentea izan dadin.

b) Aurreko ataleko integral parametrikoak definituriko *F* funtzioa kontuan hartuta, aurkitu $\int_1^\infty x^2 \cdot e^{-x} dx$ integral konbergentearen balioa integrazio-metodorik erabili gabe.

(2 puntu)

a) Integral inpropioaren puntu singular bakarra ∞ da.

Baldintza beharrezkoa: $\lim_{x\to\infty} e^{\lambda x} = \begin{cases} 0 & \forall\lambda < 0 \\ \infty & \forall\lambda > 0 \end{cases} \Rightarrow$ dibergentea da.

Eta $\forall\lambda < 0$ integrala kalkulatuko dugu:

$$F(\lambda) = \int_1^\infty e^{\lambda x} dx = \left.\frac{e^{\lambda x}}{\lambda}\right|_1^\infty = \frac{1}{\lambda}\left(\lim_{x\to\infty} e^{\lambda x} - e^{\lambda}\right) = -\frac{e^{\lambda}}{\lambda} \in \mathbb{R} \quad \Rightarrow \text{ konbergentea da.}$$

b) $\forall\lambda < 0 \quad F(\lambda) = \int_1^\infty e^{\lambda x} dx = -\frac{e^{\lambda}}{\lambda}$ integral parametriko inpropio konbergentea λ-rekiko deriba dezakegu:

$$\forall\lambda < 0 \quad F'(\lambda) = \int_1^\infty x \cdot e^{\lambda x} dx = \frac{-\lambda \cdot e^{\lambda} + e^{\lambda}}{\lambda^2} = e^{\lambda} \cdot \frac{1-\lambda}{\lambda^2}$$

Eta berriro λ-rekiko deribatuz:

$$\forall\lambda < 0 \quad F''(\lambda) = \int_1^\infty x^2 \cdot e^{\lambda x} dx = e^{\lambda} \cdot \frac{1-\lambda}{\lambda^2} + e^{\lambda} \cdot \frac{-\lambda^2 - 2\lambda(1-\lambda)}{\lambda^4} = -e^{\lambda} \cdot \frac{\lambda^2 - 2\lambda + 2}{\lambda^3}$$

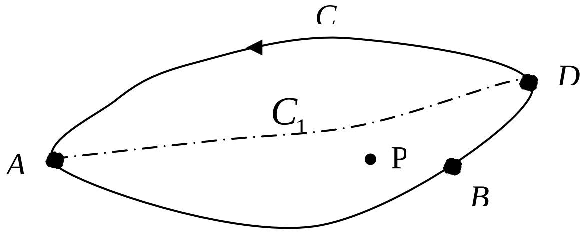
Baldin $\lambda = -1$

$$F''(-1) = \int_1^\infty x^2 \cdot e^{-x} dx = -e^{-1} \cdot \frac{1+2+2}{-1} = \frac{5}{e}$$

5.- a) Kalkulatu $C \equiv xy^2 - 2x^2 + 3y = 0$ **kurban zehar** $\int_A^B \left((1+y)dx + xdy\right)$**, non** $A = (0,0)$ **eta** $B = (3,2)$**.**

b) Har dezagun $\vec{F}(x,y) = X(x,y)\vec{i} + Y(x,y)\vec{j}$ **eremu bektorial jarraitua deribatu partzial jarraituekin** $\mathbb{R}^2 - \{P\}$ **multzoan, non** $X'_y = Y'_x$ **egiaztatzen duen.**

Izan bitez grafikoan erakusten diren *A*, *B* eta *D* puntuak eta *C* eta C_1 **kurbak:**



$\oint_C \vec{F}\cdot d\vec{r} = 9$**,** $\int_{C_1(A\to D)} \vec{F}\cdot d\vec{r} = 5$ **eta** $\int_{C(A\to B)} \vec{F}\cdot d\vec{r} = 8$ **ezagutuz gero, kalkulatu** $\int_{C(D\to B)} \vec{F}\cdot d\vec{r}$

(2 puntu)

a) $\int_A^B \left((1+y)dx + xdy\right) = \int_A^B \vec{F}\cdot d\vec{r}$

$\vec{F} = X\cdot\vec{i} + Y\cdot\vec{j} = (1+y)\cdot\vec{i} + x\cdot\vec{j}$ eta bere deribatu partzialak jarraituak dira $\mathbb{R}^2$ eremuan non $X'_y = 1 = Y'_x$. Beraz, $\int_A^B \vec{F}\cdot d\vec{r}$ bidearekiko independentea da. Orduan, emandako *C* kurba gainetik integratu beharrean, *A* eta *B* puntuak elkartzen dituen bide zuzena aukeratuko dugu lerro-integrala kalkulatzeko:

$$C' \equiv y = \frac{2x}{3} \quad\Rightarrow\quad dy = \frac{2}{3}dx \quad\Rightarrow\quad \int_A^B \left((1+y)dx + xdy\right) = \int_0^3 \left(\left(1+\frac{2x}{3}\right) + \frac{2}{3}x\right)dx =$$

$$= \int_0^3 \left(1 + \frac{4x}{3}\right)dx = x + \frac{2x^2}{3}\bigg|_0^3 = 3 + 6 = 9$$

Edo funtzio potentziala ere erabil genezake:

$$U(x,y) = \int_0^x (1+y)dt + \int_0^y 0dt + k = (1+y)x + k \quad\Rightarrow\quad \int_A^B \vec{F}\cdot d\vec{r} = U(B) - U(A) = (1+2)\cdot 3 = 9$$

b) $\oint_C \vec{F}\cdot d\vec{r} = 9 \quad\Leftrightarrow\quad \oint_C \vec{F}\cdot d\vec{r} = -9$

$\int_{C(A\to B)} \vec{F}\cdot d\vec{r} = 8 \quad\Leftrightarrow\quad \int_{C(B\to A)} \vec{F}\cdot d\vec{r} = -8$

Eta eremu bikoizki konexuetarako teoremetatik:

$$\oint_C \vec{F}\cdot d\vec{r} = \int_{C_1(A\to D)} \vec{F}\cdot d\vec{r} + \int_{C(D\to B)} \vec{F}\cdot d\vec{r} + \int_{C(B\to A)} \vec{F}\cdot d\vec{r}$$

Orduan: $\int_{C(D\to B)} \vec{F}\cdot d\vec{r} = \oint_C \vec{F}\cdot d\vec{r} - \int_{C_1(A\to D)} \vec{F}\cdot d\vec{r} - \int_{C(B\to A)} \vec{F}\cdot d\vec{r} = -9 - 5 + 8 = -6$

6.- Izan bedi $S_1 \equiv x^2+y^2+z^2=5$, $z \geq 0$ izanik, eta $S_2 \equiv (z+1)^2 = x^2+y^2$, $z \geq -1$ izanik, gainazalek osaturiko *S* gainazal itxiak mugatzen duen *V* solidoa. Har dezagun, ere, $\vec{F}(x,y,z) = x\cdot\vec{i} + z\cdot\vec{j} + (y-z)\cdot\vec{k}$ eremu bektoriala.

a) Kalkulatu *V*-ren bolumena.

b) Aurkitu *S* gainazala osatzen duen S_2 gainazalaren zatiaren azalera.

c) Kalkulatu $\vec{F}$ bektoreari dagozkion fluxuak (irteten direnak):

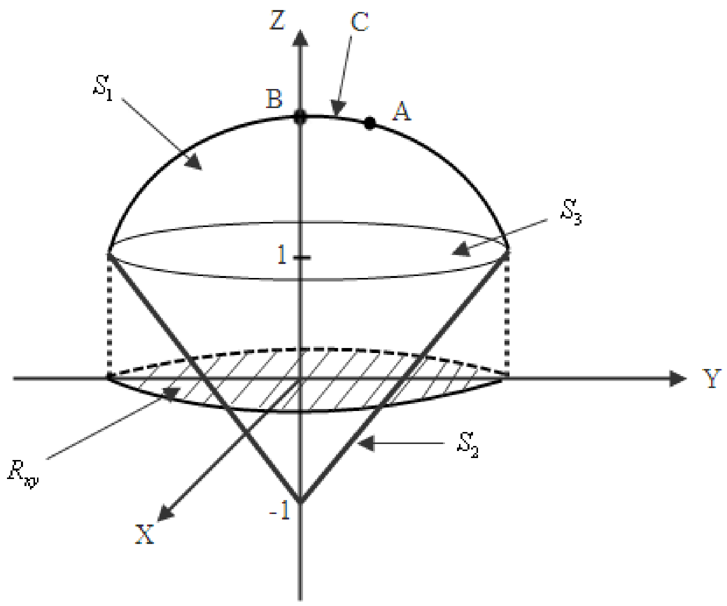
i. *S* gainazalean zehar.

ii. *S* gainazala osatzen duen S_1 gainazalaren zatian zehar.

d) Kalkulatu $\vec{F}$ bektorearen lerro-integrala $C \equiv \begin{cases} y^2+z^2=5 \\ x=0 \end{cases}$ kurban zehar,

$A = \left(0, \frac{\sqrt{15}}{2}, \frac{\sqrt{5}}{2}\right)$ **puntutik,** $B = \left(0,0,\sqrt{5}\right)$ **puntura.**

(4 puntu)



a) $\text{Bolumena}(V) = \iiint\limits_V dxdydz$

$$S_1 \cap S_2 \quad \Rightarrow \quad (z+1)^2+z^2=5 \quad \Leftrightarrow \quad 2z^2+2z-4=0 \quad \Leftrightarrow \quad z^2+z-2=0 \quad \Rightarrow \quad \begin{cases} z=1 \\ z=-2<-1\# \end{cases}$$

Beraz,

$$S_1 \equiv z = \sqrt{5-x^2-y^2} \quad \forall (x,y) \in R_{xy} \equiv x^2+y^2 \leq 4$$

$$S_2 \equiv z = -1+\sqrt{x^2+y^2} \quad \forall (x,y) \in R_{xy} \equiv x^2+y^2 \leq 4$$

Eta zilindrikoetan adierazita:

$$\begin{cases} x=\rho\cos\theta \\ y=\rho\sin\theta \\ z=z \end{cases} \Rightarrow \quad |J| = \rho \quad \text{eta} \quad V \equiv \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq \rho \leq 2 \\ -1+\rho \leq z \leq \sqrt{5-\rho^2} \end{cases} \quad \Rightarrow$$

$$\Rightarrow \quad \text{Bolumena}(V)=\int_0^{2\pi}\int_0^2\int_{\rho-1}^{\sqrt{5-\rho^2}}\rho dz d\rho d\theta=\int_0^{2\pi}\int_0^2\rho\left(\sqrt{5-\rho^2}-\rho+1\right)d\rho d\theta=$$

$$=2\pi\left[-\frac{\left(5-\rho^2\right)^{3/2}}{3}-\frac{\rho^3}{3}+\frac{\rho^2}{2}\right]_0^2=2\pi\left[-\frac{1}{3}-\frac{8}{3}+2+\frac{5^{3/2}}{3}\right]=\frac{2\pi}{3}\left(5^{3/2}-3\right)$$

b) $\text{Azalera}\left(S_2\right)=\iint_{S_2}dS=\iint_{R_{xy}}\left|\vec{N}\right|dxdy=\iint_{R_{xy}}\sqrt{1+\left(z'_x\right)^2+\left(z'_y\right)^2}dxdy$

non $\begin{cases}\vec{N}=\left(-z'_x,-z'_y,1\right)\perp S_2\\ S_2\equiv z=-1+\sqrt{x^2+y^2}\quad \forall(x,y)\in R_{xy}\equiv x^2+y^2\le 4\end{cases}\Rightarrow\begin{cases}z'_x=\dfrac{x}{\sqrt{x^2+y^2}}\\ z'_y=\dfrac{y}{\sqrt{x^2+y^2}}\end{cases}\Rightarrow$

$$\Rightarrow\quad\left|\vec{N}\right|=\sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}=\sqrt{2}$$

Orduan:

$$\text{Azalera}\left(S_2\right)=\iint_{R_{xy}}\sqrt{2}dxdy=\sqrt{2}\cdot\text{Azalera}\left(R_{xy}\right)=4\pi\sqrt{2}$$

c) i) $\Phi_S\left(\vec{F}\right)\overset{(S\text{ itxia}\Rightarrow\text{GAUSS})}{=}\iiint_V\underbrace{div\left(\vec{F}\right)}_{=0}dxdydz=0$

ii) <u>Bi modutan</u> kalkula daiteke:

- Dagokion gainazal-integrala ebatziz:

$$\Phi_{S_1}\left(\vec{F}\right)=\iint_{S_1}\vec{F}\cdot d\vec{S}=\iint_{S_1}\left(xdydz+zdzdx+(y-z)dxdy\right)=\pm\iint_{R_{xy}}\left(\vec{F}\cdot\vec{N}\right)dxdy=$$

non $\begin{cases}\vec{N}=\left(-z'_x,-z'_y,1\right)\perp S_1\\ S_1\equiv z=\sqrt{5-x^2-y^2}\quad\forall(x,y)\in R_{xy}\equiv x^2+y^2\le 4\end{cases}\Rightarrow\begin{cases}z'_x=\dfrac{-x}{\sqrt{5-x^2-y^2}}\\ z'_y=\dfrac{-y}{\sqrt{5-x^2-y^2}}\end{cases}$

$$=\pm\iint_{R_{xy}}\left(\frac{x^2}{\sqrt{5-x^2-y^2}}+y+y-\sqrt{5-x^2-y^2}\right)dxdy=$$

Polarretan adierazita:

$\begin{cases}x=\rho\cos\theta\\ y=\rho\sin\theta\end{cases}\Rightarrow\quad |J|=\rho\quad\text{eta}\quad\begin{cases}0\le\theta\le 2\pi\\ 0\le\rho\le 2\end{cases}$

$$\underset{\left(\gamma<\frac{\pi}{2}\right)}{=}\int_0^{2\pi}\int_0^2\left(\frac{\rho^2\cos^2\theta}{\sqrt{5-\rho^2}}+2\rho\sin\theta-\sqrt{5-\rho^2}\right)\rho\, d\rho d\theta=$$

$$= \int_0^{2\pi}\int_0^2 \frac{\rho^3\cos^2\theta}{\sqrt{5-\rho^2}}\,d\rho d\theta + \int_0^{2\pi}\int_0^2 2\rho^2\sin\theta\,d\rho d\theta - \int_0^{2\pi}\int_0^2 \rho\sqrt{5-\rho^2}\,d\rho d\theta = I_1 + I_2 + I_3$$

$$I_1 = \int_0^{2\pi}\int_0^2 \frac{\rho^3\cos^2\theta}{\sqrt{5-\rho^2}}\,d\rho d\theta = \int_0^{2\pi}\int_0^2 \cos^2\theta\cdot\rho^2\,\frac{\rho}{\sqrt{5-\rho^2}}\,d\rho d\theta \overset{(*)}{=}$$

$$= \int_0^{2\pi}\cos^2\theta\left[-\rho^2\sqrt{5-\rho^2}\Big|_0^2 + 2\int_0^2\rho\sqrt{5-\rho^2}\,d\rho\right]d\theta = \int_0^{2\pi}\cos^2\theta\left[-4+2\int_0^2\rho\sqrt{5-\rho^2}\,d\rho\right]d\theta =$$

$$= \int_0^{2\pi}\cos^2\theta\left[-4-\left.\frac{\left(5-\rho^2\right)^{3/2}}{3/2}\right|_0^2\right]d\theta = \int_0^{2\pi}\cos^2\theta\left[-4-\frac{2}{3}+\frac{2}{3}5^{3/2}\right]d\theta = \frac{10\sqrt{5}-14}{3}\int_0^{2\pi}\cos^2\theta d\theta =$$

$$= \frac{10\sqrt{5}-14}{3}\left(\frac{\theta}{2}+\frac{\sin(2\theta)}{4}\right)\Bigg|_0^{2\pi} \boxed{= \frac{10\sqrt{5}-14}{3}\pi = I_1}$$

(*) Zatikako integrazioa erabiliz: $\begin{cases} u=\rho^2 \quad \Rightarrow \quad du = 2\rho d\rho \\ dv = \dfrac{\rho}{\sqrt{5-\rho^2}}\,d\rho \quad \Rightarrow \quad v = -\sqrt{5-\rho^2} \end{cases}$

$$I_2 = \int_0^{2\pi}\int_0^2 2\rho^2\sin\theta\,d\rho d\theta = \frac{16}{3}\int_0^{2\pi}\sin\theta\,d\theta = -\frac{16}{3}\cos\theta\Big|_0^{2\pi} \boxed{= 0 = I_2}$$

$$I_3 = -\int_0^{2\pi}\int_0^2 \rho\sqrt{5-\rho^2}\,d\rho d\theta = -2\pi\int_0^2\rho\sqrt{5-\rho^2}\,d\rho = \pi\left.\frac{\left(5-\rho^2\right)^{3/2}}{3/2}\right|_0^2 \boxed{= \frac{2\pi}{3}\left(1-5\sqrt{5}\right) = I_3}$$

Eta azken hiru emaitza hauek batuz:

$$\Phi_{S_1}\left(\vec{F}\right) = I_1 + I_2 + I_3 = \frac{10\sqrt{5}-14}{3}\pi + \frac{2\pi}{3}\left(1-5\sqrt{5}\right) = \frac{\pi}{3}\left(10\sqrt{5}-14+2-10\sqrt{5}\right) = \frac{-12\pi}{3} = -4\pi$$

- Beste gainazal itxia definituz: $S' = S_1 \bigcup S_3$, non $S_3 \equiv z = 1 \quad \forall(x,y)\in R_{xy}$.

Honela,

$$\Phi_{S'}\left(\vec{F}\right) \overset{(S' \text{ itxia} \Rightarrow \text{GAUSS})}{=} 0 = \Phi_{S_1}\left(\vec{F}\right) + \Phi_{S_3}\left(\vec{F}\right) \quad \Leftrightarrow \quad \Phi_{S_1}\left(\vec{F}\right) = -\Phi_{S_3}\left(\vec{F}\right)$$

Eta $\Phi_{S_3}\left(\vec{F}\right) = \iint_{S_3}\vec{F}\cdot d\vec{S} = \iint_{S_3}\left(xdyxz + zdzdx + (y-z)dxdy\right) \underset{(S_3\equiv z=1\Rightarrow dz=0)}{=} \pm\iint_{R_{xy}}(y-1)dxdy =$

$$\underset{(\gamma>\frac{\pi}{2})}{=} -\iint_{R_{xy}}(y-1)dxdy = \iint_{R_{xy}}(1-y)dxdy = \iint_{R_{xy}}dxdy - \underbrace{\iint_{R_{xy}}ydxdy}_{\substack{=0 \\ y \text{ bakoitia} \\ R_{xy} \text{ simetrikoa} \\ y=0 \text{ zuzenarekiko}}} = \text{Azalera}\left(R_{xy}\right) = 4\pi$$

Beraz, $\Phi_{S_1}\left(\vec{F}\right) = -4\pi$

(Argi dago zein den garapenik egokiena)

d) $\int_A^B \vec{F}\cdot d\vec{r} = \int_A^B \left(xdx + zdy + (y-z)dz\right) \quad C \equiv \begin{cases} y^2+z^2=5 \\ x=0 \end{cases}$ kurban zehar.

Hau ere <u>bi eratan</u> egin daiteke:

- Emandako bidea jarraituz:

$$C \equiv \begin{cases} y^2+z^2=5 \\ x=0 \end{cases} \equiv \begin{cases} x=0 \\ y=\sqrt{5}\cos t \\ z=\sqrt{5}\sin t \end{cases} \quad \begin{cases} A \equiv z = \dfrac{\sqrt{5}}{2} = \sqrt{5}\sin t \quad \Leftrightarrow \quad t=\dfrac{\pi}{6}\text{etik} \\ B \equiv z = \sqrt{5} = \sqrt{5}\sin t \quad \Leftrightarrow \quad t=\dfrac{\pi}{2}\text{era} \end{cases}$$

Orduan:

$$\int_A^B \vec{F}\cdot d\vec{r} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(-5\sin^2 t + 5(\cos t - \sin t)\cos t\right)dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(5\left(\cos^2 t - \sin^2 t\right) - 5\sin t\cos t\right)dt$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(5\cos(2t) - 5\sin t\cos t\right)dt = \frac{5}{2}\sin(2t) - \frac{5}{2}\sin^2 t\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = -\frac{5}{2} - \frac{5}{2}\cdot\frac{\sqrt{3}}{2} + \frac{5}{2}\cdot\frac{1}{4} = -\frac{15+10\sqrt{3}}{8}$$

- $\vec{F}(x,y,z) = x\cdot\vec{i} + z\cdot\vec{j} + (y-z)\cdot\vec{k}$ eta bere deribatu partzialak jarraituak dira $\mathbb{R}^3$ eremuan non $\begin{cases} X'_y = 0 = Y'_x \\ X'_z = 0 = Z'_x \\ Y'_z = 1 = Z'_y \end{cases}$. Orduan $\int_A^B \vec{F}\cdot d\vec{r}$ bidearekiko independentea da. Beraz, *A* eta *B* puntuak elkartzen dituen bide zuzenean zehar integratuko dugu:

$$C' \equiv \begin{cases} x=0 \\ z=\sqrt{5}-\dfrac{y}{\sqrt{3}} \end{cases} \Rightarrow \begin{cases} dx=0 \\ dz=-\dfrac{1}{\sqrt{3}}dy \end{cases} \quad y\ \frac{\sqrt{15}}{2}\text{etik 0ra} \quad \Rightarrow$$

$$\int_A^B \vec{F}\cdot d\vec{r} = \int_{\frac{\sqrt{15}}{2}}^{0} \left(\sqrt{5} - \frac{y}{\sqrt{3}} - \frac{1}{\sqrt{3}}\left(y - \sqrt{5} + \frac{y}{\sqrt{3}}\right)\right)dz = \left(\sqrt{5} + \frac{\sqrt{5}}{\sqrt{3}}\right)y - \frac{y^2}{2}\left(\frac{2}{\sqrt{3}} + \frac{1}{3}\right)\Bigg|_{\frac{\sqrt{15}}{2}}^{0} =$$

$$= -\left(\sqrt{5} + \frac{\sqrt{5}}{\sqrt{3}}\right)\frac{\sqrt{15}}{2} + \frac{15}{8}\left(\frac{2}{\sqrt{3}} + \frac{1}{3}\right) = -\frac{10\sqrt{3}+15}{8}$$

Edo, horren ordez, funtzio potentziala ere erabil genezake:

$$U(x,y,z) = \int_0^x tdt + \int_0^y zdt - \int_0^z tdt + k = \frac{x^2}{2} + yz - \frac{z^2}{2} + k$$

Eta, $\int_A^B \vec{F}\cdot d\vec{r} = U(B) - U(A) = U\left(0,0,\sqrt{5}\right) - U\left(0, \dfrac{\sqrt{15}}{2}, \dfrac{\sqrt{5}}{2}\right) = -\dfrac{5}{2} - \dfrac{5\sqrt{3}}{4} + \dfrac{5}{8} = -\dfrac{15+10\sqrt{3}}{8}$