KALKULUA – MINTEGIETAKO 2. KONTROLA (A eredua)

IZEN-ABIZENAK:
TALDEA:

1.- $f(x)=\begin{cases}\dfrac{\left(e^x-1\right)\cdot\arcsin\left(x^2\right)}{x^2} & \forall x<0\\ 0 & x=0\\ \sqrt{x}\cdot\sin\left(\sqrt{x}\right) & \forall x>0\end{cases}$ **funtzioa emanik, kalkulatu** $f'(0)$**.**

(2 puntu)

$$f'(0^-)=\lim_{h\to 0^-}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^-}\frac{\dfrac{\left(e^h-1\right)\cdot\arcsin\left(h^2\right)}{h^2}}{h}=\lim_{h\to 0^-}\frac{\left(e^h-1\right)\cdot\arcsin\left(h^2\right)}{h\cdot h^2}=$$

$$=\lim_{h\to 0^-}\frac{L\left(e^h\right)\cdot h^2}{h^3}=\lim_{h\to 0^-}\frac{h\cdot h^2}{h^3}=1$$

$$f'(0^+)=\lim_{h\to 0^+}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^+}\frac{\sqrt{h}\cdot\sin\left(\sqrt{h}\right)}{h}=\lim_{h\to 0^+}\frac{\sin\left(\sqrt{h}\right)}{\sqrt{h}}=1$$

Beraz, $f'(0^+)=f'(0^-)=1 \quad\Leftrightarrow\quad f'(0)=1$

2.- $f(x)=\begin{cases}x^2+x^4\cdot\sin\left(\dfrac{1}{x}\right) & \forall x\neq 0\\ 0 & x=0\end{cases}$ **funtzioa emanik, kalkulatu** $f'(0)$ **eta** $f''(0)$

(2 puntu)

$$f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^-}\frac{h^2+h^4\cdot\sin\left(\dfrac{1}{h}\right)}{h}=\lim_{h\to 0^-}\left(h+h^3\cdot\sin\left(\frac{1}{h}\right)\right)=0$$

Eta $\forall x\neq 0 \quad f'(x)=2x+4x^3\cdot\sin\left(\dfrac{1}{x}\right)-x^2\cdot\cos\left(\dfrac{1}{x}\right)$

$$f''(0)=\lim_{h\to 0}\frac{f'(h)-f'(0)}{h}=\lim_{h\to 0^-}\frac{2h+4h^3\cdot\sin\left(\dfrac{1}{h}\right)-h^2\cdot\cos\left(\dfrac{1}{h}\right)}{h}=$$

$$=\lim_{h\to 0^-}\left(2+4h^2\cdot\sin\left(\frac{1}{h}\right)-h\cdot\cos\left(\frac{1}{h}\right)\right)=2$$

KALKULUA – MINTEGIETAKO 2. KONTROLA (B eredua)

IZEN-ABIZENAK:
TALDEA:

1.- $f(x)=\begin{cases} \dfrac{\arctan(2x)\cdot\sin^2 x}{x^2} & \forall x<0 \\ 0 & x=0 \\ 3^{\sqrt{x}}\cdot\sin x & \forall x>0 \end{cases}$ **funtzioa emanik, kalkulatu** $f'(0)$**.**

(2 puntu)

$$f'(0^-)=\lim_{h\to 0^-}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^-}\frac{\dfrac{\arctan(2h)\cdot\sin^2 h}{h^2}}{h}=\lim_{h\to 0^-}\frac{\arctan(2h)\cdot\sin^2 h}{h\cdot h^2}=\lim_{h\to 0^-}\frac{2h\cdot h^2}{h\cdot h^2}=2$$

$$f'(0^+)=\lim_{h\to 0^+}\frac{f(h)-f(0)}{h}=\lim_{h\to 0^+}\frac{3^{\sqrt{h}}\cdot\sin h}{h}=1$$

Beraz, $f'(0^-)=2\neq f'(0^+)=1 \quad\Rightarrow\quad \nexists\ f'(0)$

2.- $f(x)=\begin{cases} x^4\cdot\cos\left(\dfrac{1}{x}\right)-3x^2 & \forall x\neq 0 \\ 0 & x=0 \end{cases}$ **funtzioa emanik, kalkulatu** $f'(0)$ **eta** $f''(0)$

(2 puntu)

$$f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{h^4\cdot\cos\left(\dfrac{1}{h}\right)-3h^2}{h}=\lim_{h\to 0}\left(h^3\cdot\cos\left(\frac{1}{h}\right)-3h\right)=0$$

Eta $\forall x\neq 0 \quad f'(x)=4x^3\cdot\cos\left(\dfrac{1}{x}\right)+x^2\cdot\sin\left(\dfrac{1}{x}\right)-6x$

$$f''(0)=\lim_{h\to 0}\frac{f'(h)-f'(0)}{h}=\lim_{h\to 0^-}\frac{4h^3\cdot\cos\left(\dfrac{1}{h}\right)+h^2\cdot\sin\left(\dfrac{1}{h}\right)-6h}{h}=$$

$$=\lim_{h\to 0^-}\left(4h^2\cdot\cos\left(\frac{1}{h}\right)+h\cdot\sin\left(\frac{1}{h}\right)-6\right)=-6$$

OHARRAK

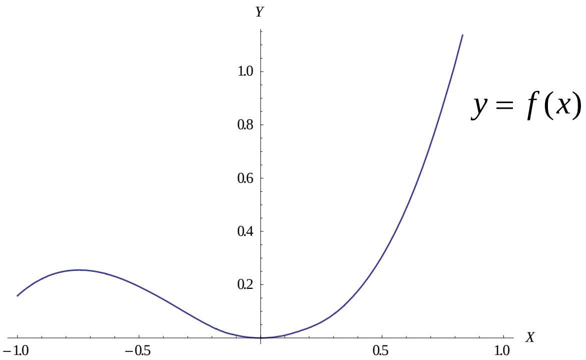
<u>A ereduko 2. ariketan</u>: $f(x)=\begin{cases} x^2+x^4\cdot\sin\left(\dfrac{1}{x}\right) & \forall x\neq 0 \\ 0 & x=0 \end{cases}$

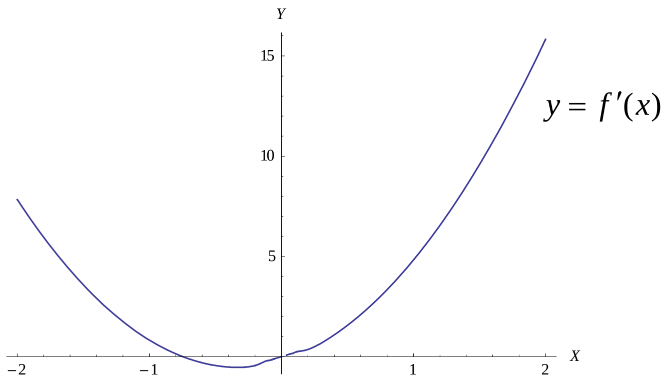
$$\forall x\neq 0 \quad f'(x)=2x+4x^3\cdot\sin\left(\frac{1}{x}\right)-x^2\cdot\cos\left(\frac{1}{x}\right) \quad \Rightarrow$$

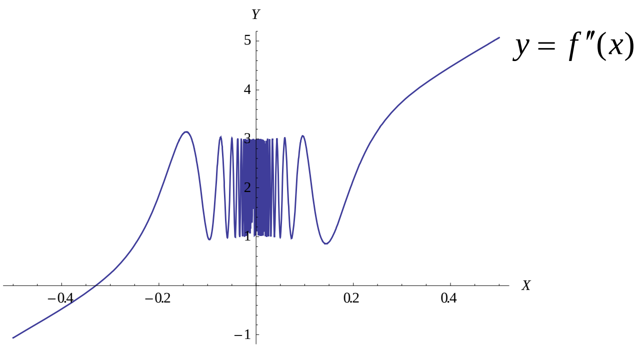
$$\Rightarrow \quad f''(x)=2+12x^2\cdot\sin\left(\frac{1}{x}\right)-6x\cdot\cos\left(\frac{1}{x}\right)-\sin\left(\frac{1}{x}\right) \quad \forall x\neq 0$$

$\lim\limits_{x\to 0} f'(x)=\lim\limits_{x\to 0}\left(2x+4x^3\cdot\sin\left(\dfrac{1}{x}\right)-x^2\cdot\cos\left(\dfrac{1}{x}\right)\right)=0=f'(0) \quad \Rightarrow \quad f'$ jarraitua da $x=0$ puntuan.

Baina $\lim\limits_{x\to 0} f''(x)=\lim\limits_{x\to 0}\left(2+12x^2\cdot\sin\left(\dfrac{1}{x}\right)-6x\cdot\cos\left(\dfrac{1}{x}\right)-\sin\left(\dfrac{1}{x}\right)\right)\nexists \quad \Rightarrow \quad f''$ ez da jarraitua $x=0$ puntuan. Hala ere, $\nexists \lim\limits_{x\to 0} f''(x) \quad \not\Rightarrow \quad \nexists f''(0)$







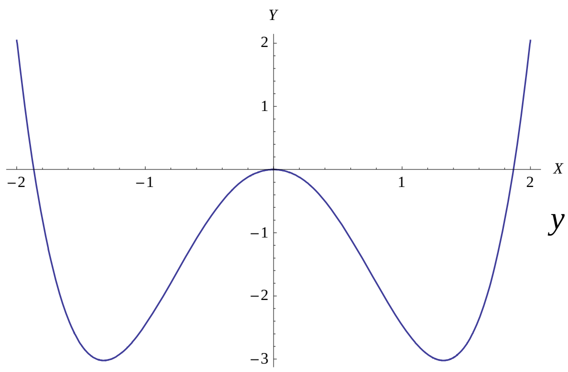
B ereduko 2. ariketan: $f(x)=\begin{cases} x^4\cdot\cos\left(\dfrac{1}{x}\right)-3x^2 & \forall x\neq 0 \\ 0 & x=0 \end{cases}$

$$\forall x\neq 0 \quad f'(x)=4x^3\cdot\cos\left(\frac{1}{x}\right)+x^2\cdot\sin\left(\frac{1}{x}\right)-6x \quad \Rightarrow$$

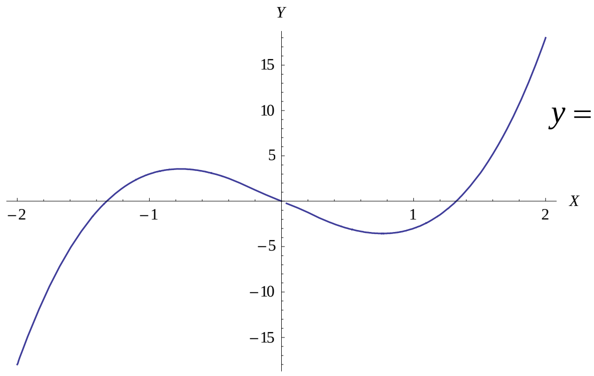
$$\Rightarrow \quad f''(x)=12x^2\cdot\cos\left(\frac{1}{x}\right)+6x\cdot\sin\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)-6 \quad \forall x\neq 0$$

$\lim\limits_{x\to 0} f'(x)=\lim\limits_{x\to 0}\left(4x^3\cdot\cos\left(\dfrac{1}{x}\right)+x^2\cdot\sin\left(\dfrac{1}{x}\right)-6x\right)=0=f'(0) \quad \Rightarrow \quad f'$ jarraitua da $x=0$ puntuan.

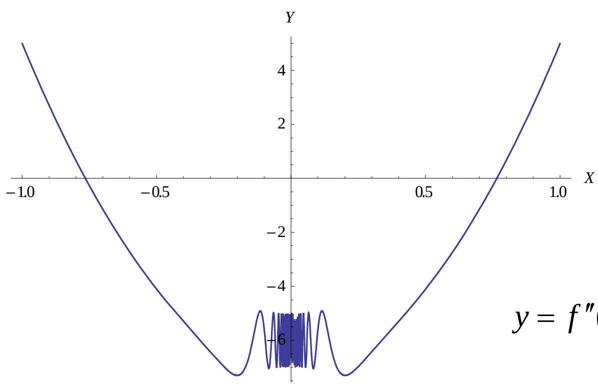
Baina $\lim\limits_{x\to 0} f''(x)=\lim\limits_{x\to 0}\left(12x^2\cdot\cos\left(\dfrac{1}{x}\right)+6x\cdot\sin\left(\dfrac{1}{x}\right)-\cos\left(\dfrac{1}{x}\right)-6\right)\nexists \quad \Rightarrow \quad f''$ ez da jarraitua $x=0$ puntuan. Hala ere, $\nexists \lim\limits_{x\to 0} f''(x) \quad \not\Rightarrow \quad \nexists f''(0)$



$y=f(x)$



$y=f'(x)$



$y=f''(x)$