Unified representability of total preorders, semiorders, and interval orders through scales and a single map

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(joint work with Esteban Induráin, UPNA)

WiAT '09 Workshop in Applied Topology

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Total preorders, semiorders and interval orders

definitions

Let X be a set and $\mathcal{R}$ a reflexive binary and complete relation on X ($\mathcal{R}$ is said to be complete if for each $x, y \in X$ either $x\mathcal{R}y$ or $y\mathcal{R}x$).

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total preorder $\Longrightarrow$ semiorder $\Longrightarrow$ interval order

Total preorders, semiorders and interval orders

examples (1)

FRIDAY 27 JUNE 2008

	Salón de Actos	Aula 0.0	Aula 0.1	Aula 0.2	Aula 0.3
10:00-10:50	**ANTONIO FERNÁNDEZ** Representation of Banach lattices				
10:50-11:20	COFFEE BREAK				
11:20-12:10	**ÁNGEL TAMARIZ-MASCARUA** Spaces of continuous functions defined on Mrówka spaces				
	Salón de Actos	**Aula 0.0**	**Aula 0.1**	**Aula 0.2**	**Aula 0.3**
12:20-12:40	**B. Requejo** Dimension on topological spaces	**J. Marín Molina** Weak bases and quasi-metrization of bispaces	**H.W. Martin** Lattices of Metrics on Cantor sets	**F. Balibrea** On Periodic-Recurrent property on Continua of low dimension	**A. González** Biorthogonal systems in WCG Banach spaces
12:40-13:00	**M. Mrsevic** Some properties of hyperspaces of Cech closure spaces	**J.M. Sánchez Álvarez** Quasi-metrics and (monotone) normality	**A. Le Donne** On metric spaces and local extrema	**J. Ferreira Alves** Zeta functions and other topological invariants for time-periodic	**V. Montesinos** On bounded biorthogonal systems
13:00-13:20	**M.J. López** Monotone and light induced maps on n-fold hyperspaces	**O. Valero** An extension of the dual complexity spaces and applications	**G. Gutierres** Totally bounded metric spaces and the Axiom of Choice	**G. Soler López** Minimal non orientable surfaces	**J. Ferrer** On a certain class of compacta separating chain conditions
13:20-13:40	**D. Herrera-Carrasco** Dendrites without unique hyperspace	**P. Tirado** Fixed point theorems in stationary fuzzy quasi- metric spaces and [0,1]-fuzzy posets	**D. Gauld** Foliations and non-metrisable manifold	**J.C. Valverde** Near a local topological equivalence when a quasi-center like point appears	**A. Kitover** Almost homeomorphisms of compact Hausdorff spaces
13:40-15:30	LUNCH				
15:30-15:50	**J. Picado** The real functions in pointfree topology	**G. Bosi** A note on continuous multi-utility representations of preorders	**L. Ruza** Separation Axioms and the Prime Spectrum of Commutative Semirings	J. S. Cánovas Topological entropy of continuous transitive real maps	J.C. Navarro Pascual Extremal structure and extension of uniformly continuous functions
15:50-16:10	**M.J. Ferreira** Complete normality on locales	**L.M. Brown** Real Dicompactifications	**A. Peña** Maximal and Minimal Spectrum of Commutative Semirings	**A. Nagar** Dynamics of the Induced Shift Map	**A. Jiménez Vargas** The lattice carrier space of little Lipschitz functions
16:10-16:30	**E. Induráin** Preorderable topologies	**L.M. Brown** Ditopological texture spaces and digital topology	**J. Vielma** On a question of Robert Gilmer	**M.V. Ferrer** Bounded Sets in Topological Groups	**L. Dubarbie** Biseparating maps between vector-valued Lipschitz function spaces
	Paraninfo				
16:40-17:10	COFFEE BREAK				
17:10-18:00	**JOSÉ VALERO** On the Kneser property for the Navier-Stokes system and the Ginzburg-Landau equation				
21:00	CONFERENCE DINNER				

Workshop in Applied Topology WiAT '09

Total preorders, semiorders and interval orders

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examples (2)

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(**Valero** $\mathcal{R}$ **Gutierres**) and (**Gutierres**$\mathcal{R}$ **Sánchez**). But $\neg$(**Valero**$\mathcal{R}$ **Sánchez**)

Total preorders, semiorders and interval orders examples (2)

FRIDAY 27 JUNE 2008 (Real)

	Salón de Actos	Aula 0.0	Aula 0.1	Aula 0.2	Aula 0.3
12:20-12:30	**B. Requejo** Dimension on topological spaces		**H.W. Martin** Lattices of Metrics on Cantor sets	**F. Balibrea** On Periodic-Recurrent property on Continua of low dimension	**A. González** Biorthogonal systems in WCG Banach spaces
12:30-12:40		**J. Marín Molina** Weak bases and quasi-metrization of bispaces			
12:40-12:50	**M. Mrsevic** Some properties of hyperspaces of Cech closure spaces		**A. Le Donne** On metric spaces and local extrema	**J. Ferreira Alves** Zeta functions and other topological invariants for time-periodic	**V. Montesinos** On bounded biorthogonal systems
12:50-13:00		**J.M. Sánchez Álvarez** Quasi-metrics and (monotone) normality			
13:00-13:10	**M.J. López** Monotone and light induced maps on n-fold hyperspaces		**G. Gutierres** Totally bounded metric spaces and the Axiom of Choice	**G. Soler López** Minimal non orientable surfaces	**J. Ferrer** On a certain class of compacta separating chain conditions
13:10-13:20		**O. Valero** An extension of the dual complexity spaces and applications			
13:20-13:30	**D. Herrera-Carrasco** Dendrites without unique hyperspace		**D. Gauld** Foliations and non-metrisable manifold	**J.C. Valverde** Near a local topological equivalence when a quasi-center like point appears	**A. Kitover** Almost homeomorphisms of compact Hausdorff spaces
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Total preorders, semiorders and interval orders

examples (3)

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Total preorders, semiorders and interval orders

examples (3)

FRIDAY 27 JUNE 2008 (Real 2)

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WiAT '09 Workshop in Applied Topology

Total preorders, semiorders and interval orders — examples (3)

FRIDAY 27 JUNE 2008 (Real 2)

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Total preorders, semiorders and interval orders

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Total preorders, semiorders and interval orders canonical examples

Total preorder: $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$. $\mathcal{R}_{t.p.} \equiv \leq_u$

$$a\mathcal{R}_{t.p.}b \iff a \leq b; \qquad a\mathcal{P}_{t.p.}b \iff a < b$$

Total preorders, semiorders and interval orders canonical examples

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In this case $\leq$ is even an order!

Total preorders, semiorders and interval orders — canonical examples

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Semiorder: $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$

$$a\mathcal{R}_{s.o.}b \iff a \leq b + 1; \qquad a\mathcal{P}_{s.o.}b \iff a < b - 1$$

Total preorders, semiorders and interval orders — canonical examples

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$\mathcal{R}_{s.o.}$ is an semiorder which fails to be a total preorder (i.e it fails to be transitive):

$$(3\mathcal{R}_{s.o.}2) \quad \text{and} \quad (2\mathcal{R}_{s.o.}1) \quad \text{but} \quad \neg(3\mathcal{R}_{s.o.}1)$$

Total preorders, semiorders and interval orders — canonical examples

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Interval order $\mathcal{Y} = \{a = (a_1, a_2) \in \overline{\mathbb{R}}^2 : a_1 \leq a_2\}$

$$a\mathcal{R}_{i.o.}b \iff a_1 \leq b_2; \qquad a\mathcal{P}_{i.o.}b \iff a_2 < b_1$$

Total preorders, semiorders and interval orders

canonical examples

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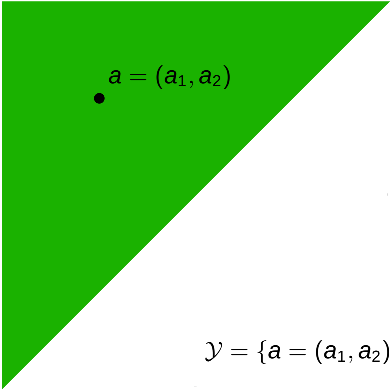
$\mathcal{R}_{i.o}$ is an interval-order which fails to be a semiorder:

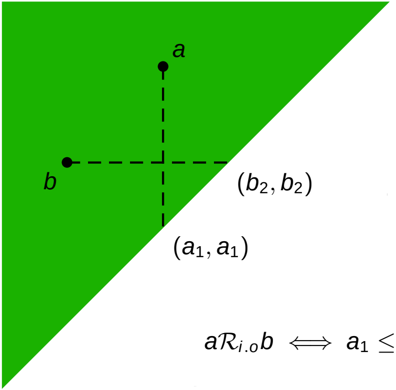
$(1,1)\mathcal{R}_{i.o}(-1,1)\mathcal{R}_{i.o}(-1,-1)$ but $(0,0)\mathcal{P}_{i.o}(1,1)$ and $(-1,-1)\mathcal{R}_{i.o}(0,0)$

Total preorders, semiorders and interval orders canonical interval order



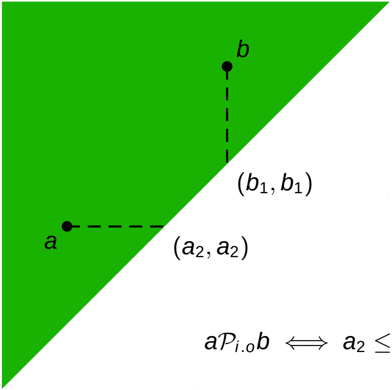
$$\mathcal{Y} = \{a = (a_1, a_2) \in \overline{\mathbb{R}}^2 : a_1 \leq a_2\}$$

Total preorders, semiorders and interval orders canonical interval order



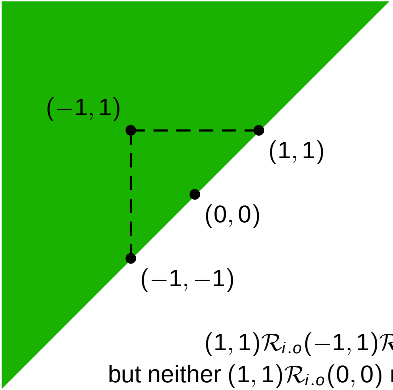
$$a\mathcal{R}_{i.o}b \iff a_1 \leq b_2$$

Total preorders, semiorders and interval orders canonical interval order



$$a\mathcal{P}_{i.o}b \iff a_2 \leq b_1$$

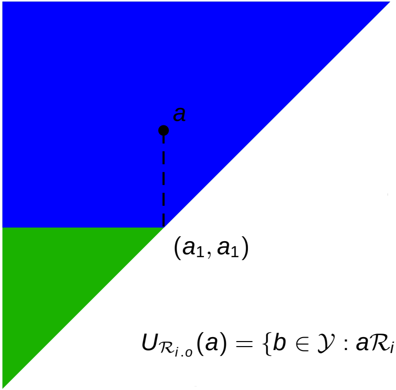
Total preorders, semiorders and interval orders canonical interval order



$(1,1)\mathcal{R}_{i.o}(-1,1)\mathcal{R}_{i.o}(-1,-1)$,
but neither $(1,1)\mathcal{R}_{i.o}(0,0)$ nor $(0,0)\mathcal{R}_{i.o}(-1,-1)$

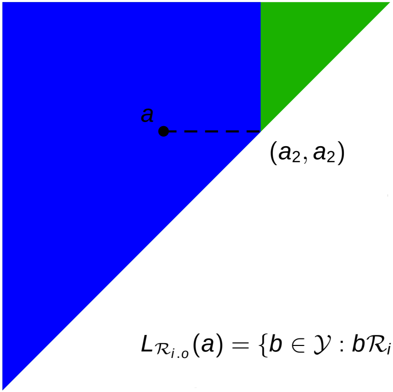
Workshop in Applied Topology WiAT '09

Total preorders, semiorders and interval orders canonical interval order



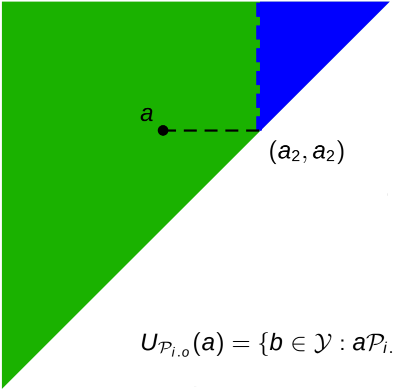
$$U_{\mathcal{R}_{i.o}}(a) = \{b \in \mathcal{Y} : a\mathcal{R}_{i.o}b\} = \{b \in \mathcal{Y} : b_2 \geq a_1\}$$

Total preorders, semiorders and interval orders canonical interval order



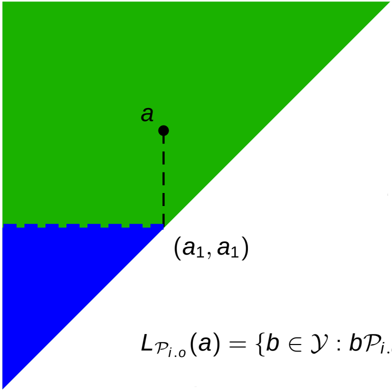
$$L_{\mathcal{R}_{i.o}}(a) = \{b \in \mathcal{Y} : b\mathcal{R}_{i.o}a\} = \{b \in \mathcal{Y} : b_1 \leq a_2\}$$

Total preorders, semiorders and interval orders — canonical interval order



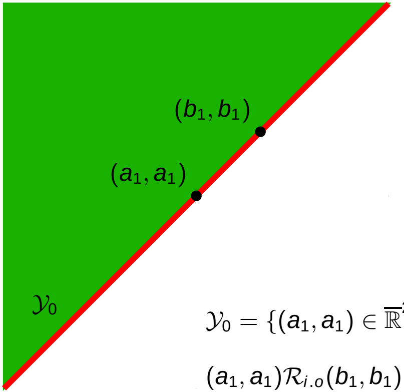
$$U_{\mathcal{P}_{i.o}}(a) = \{b \in \mathcal{Y} : a\mathcal{P}_{i.o}b\} = \{b \in \mathcal{Y} : b_1 > a_2\}$$

Total preorders, semiorders and interval orders canonical interval order



$$L_{\mathcal{P}_{i.o}}(a) = \{b \in \mathcal{Y} : b\mathcal{P}_{i.o}a\} = \{b \in \mathcal{Y} : b_2 < a_1\}$$

Total preorders, semiorders and interval orders canonical interval order



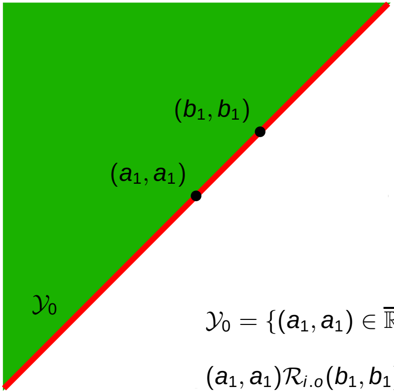
$$\mathcal{Y}_0 = \{(a_1, a_1) \in \overline{\mathbb{R}}^2 : a \in \mathbb{R}\}$$

$$(a_1, a_1)\mathcal{R}_{i.o}(b_1, b_1) \iff a_1 \leq b_1$$

$$(a_1, a_1)\mathcal{P}_{i.o}(b_1, b_1) \iff a_1 < b_1$$

Total preorders, semiorders and interval orders canonical interval order

$(\mathcal{Y}_0, \mathcal{R}_{i.o})$ is isomorphic to $(\overline{\mathbb{R}}, \leq)$



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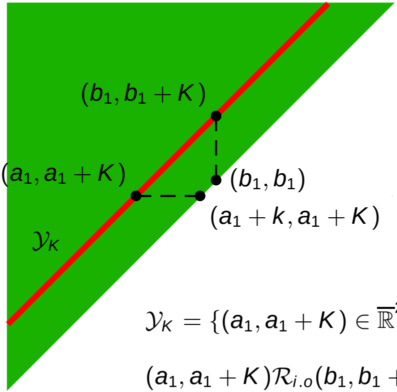
Total preorders, semiorders and interval orders canonical interval order



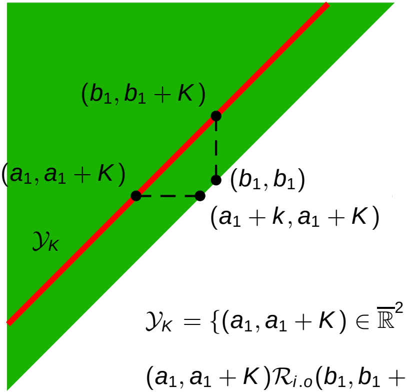
$$\mathcal{Y}_K = \{(a_1, a_1 + K) \in \overline{\mathbb{R}}^2 : a_1 \in \mathbb{R}\}$$

$$(a_1, a_1 + K)\mathcal{R}_{i.o}(b_1, b_1 + K) \iff a_1 \leq b_1 + K$$
$$(a_1, a_1 + K)\mathcal{P}_{i.o}(b_1, b_1 + K) \iff a_1 + K < b_1$$

Total preorders, semiorders and interval orders

canonical interval order

$(\mathcal{Y}_K, \mathcal{R}_{i.o})$ is isomorphic to $(\overline{\mathbb{R}}, \mathcal{R}_{s.o})$



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Representability

(1) A total preorder $\mathcal{R}$ on (X, τ) is *(continuously) representable* if there exists a (continuous) $u : (X, \tau, \mathcal{R}) \to (\mathbb{R}, \tau_u, \leq)$ such that

$$x \mathcal{R} y \iff u(x) \leq u(y) \quad (x, y \in X).$$

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(2) A semiorder $\mathcal{R}$ on (X, τ) is *(continuously) representable in* $\overline{\mathbb{R}}$ if there exist a (continuous) $u : (X, \tau, \mathcal{R}) \to (\overline{\mathbb{R}}, \tau_u)$ and $K \geq 0$ (*"discrimination threshold"*) such that

$$x\mathcal{R}y \iff u(x) \leq u(y) + K \ \ (x, y \in X).$$

Representability

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(2) A semiorder $\mathcal{R}$ on (X, τ) is *(continuously) representable in* $\overline{\mathbb{R}}$ if there exist a (continuous) $u : (X, \tau, \mathcal{R}) \to (\overline{\mathbb{R}}, \tau_u)$ such that

$$x\mathcal{R}y \iff u(x) \leq u(y) + 1 \;\; (x, y \in X).$$

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(1) A total preorder $\mathcal{R}$ on (X, τ) is *(continuously) representable* if there exists a (continuous) $u : (X, \tau, \mathcal{R}) \to (\mathbb{R}, \tau_u, \leq)$ such that

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(2) A semiorder $\mathcal{R}$ on (X, τ) is *(continuously) representable in* $\overline{\mathbb{R}}$ if there exist a (continuous) $u : (X, \tau, \mathcal{R}) \to (\overline{\mathbb{R}}, \tau_u, \mathcal{R}_{s.o})$ such that

$$x\mathcal{R}y \iff u(x) \leq u(y) + 1 \ \ (x, y \in X).$$

(3) An interval order $\mathcal{R}$ on X is said to be *(continuously) representable* if there exists a pair of (continuous) $u, v : (X, \tau) \to (\mathbb{R}, \tau_u)$ such that

$$x\mathcal{R}y \iff u(x) \leq v(y) \ \ (x, y \in X).$$

Representability

example (total preorder)

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13:20-13:40	**D. Herrera-Carrasco** Dendrites without unique hyperspace	**P. Tirado** Fixed point theorems in stationary fuzzy quasi- metric spaces and [0,1]-fuzzy posets	**D. Gauld** Foliations and non-metrisable manifold	**J.C. Valverde** Near a local topological equivalence when a quasi-center like point appears	**A. Kitover** Almost homeomorphisms of compact Hausdorff spaces

Representability

example (total preorder)

13:20-13:40:
Herrera, **Tirado**,
Gauld, **Valverde**, **Kitover**

13:00-13:20:
López, **Valero**,
Gutierres, **Soler**, **Ferrer**

12:40-13:00:
Mrsevic, **Sánchez**,
Donne, **Ferreira**, **Montesinos**

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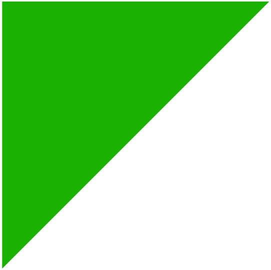

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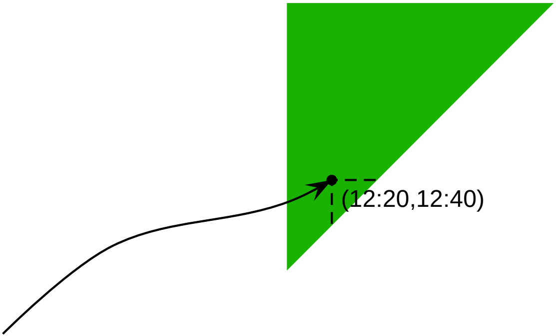

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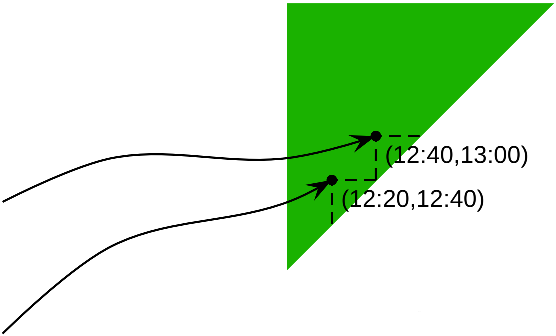

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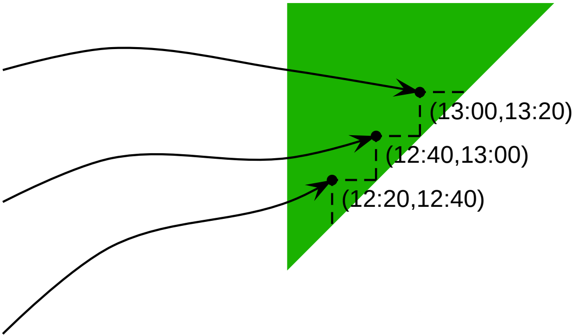

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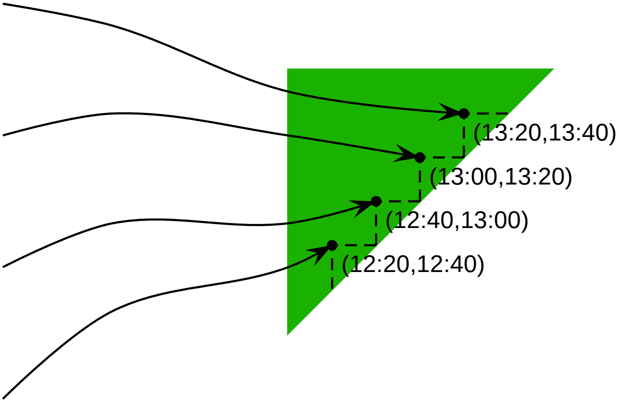

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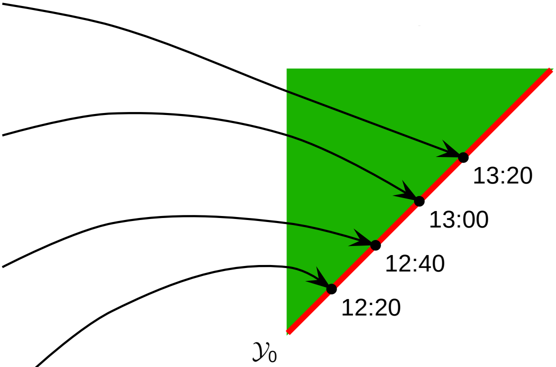

Representability

example (semiorder)

FRIDAY 27 JUNE 2008 (Real)

	Salón de Actos	Aula 0.0	Aula 0.1	Aula 0.2	Aula 0.3
12:20-12:30	**B. Requejo** Dimension on topological spaces		**H.W. Martin** Lattices of Metrics on Cantor sets	**F. Balibrea** On Periodic-Recurrent property on Continua of low dimension	**A. González** Biorthogonal systems in WCG Banach spaces
12:30-12:40		**J. Marín Molina** Weak bases and quasi-metrization of bispaces			
12:40-12:50	**M. Mrsevic** Some properties of hyperspaces of Cech closure spaces		**A. Le Donne** On metric spaces and local extrema	**J. Ferreira Alves** Zeta functions and other topological invariants for time-periodic	**V. Montesinos** On bounded biorthogonal systems
12:50-13:00		**J.M. Sánchez Álvarez** Quasi-metrics and (monotone) normality			
13:00-13:10	**M.J. López** Monotone and light induced maps on n-fold hyperspaces		**G. Gutierres** Totally bounded metric spaces and the Axiom of Choice	**G. Soler López** Minimal non orientable surfaces	**J. Ferrer** On a certain class of compacta separating chain conditions
13:10-13:20		**O. Valero** An extension of the dual complexity spaces and applications			
13:20-13:30	**D. Herrera-Carrasco** Dendrites without unique hyperspace		**D. Gauld** Foliations and non-metrisable manifold	**J.C. Valverde** Near a local topological equivalence when a quasi-center like point appears	**A. Kitover** Almost homeomorphisms of compact Hausdorff spaces
13:30-13:40		**P. Tirado** Fixed point theorems in stationary fuzzy quasi- metric spaces and [0,1]-fuzzy posets			
13:40-13:50					

Representability

example (semiorder)

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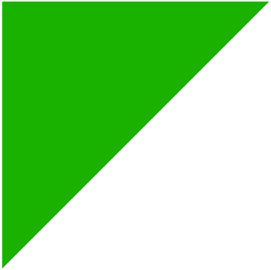

Representability

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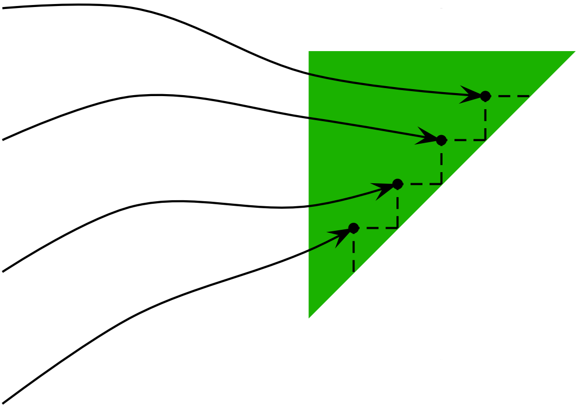

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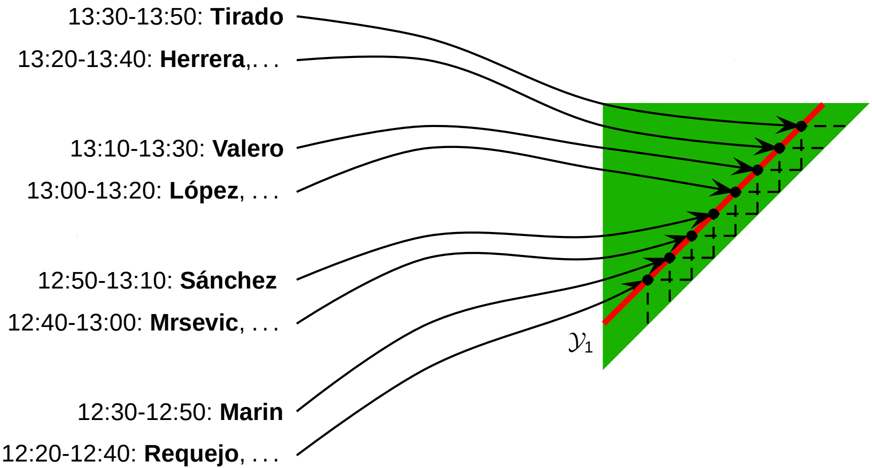
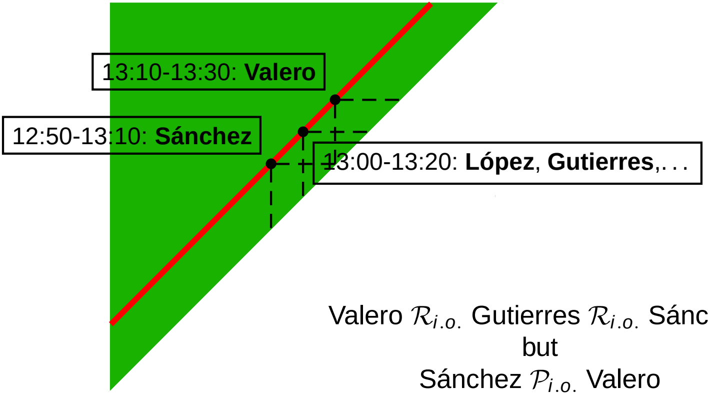
example (semiorder)



Representability

example (semiorder)



Valero $\mathcal{R}_{i.o.}$ Gutierres $\mathcal{R}_{i.o.}$ Sánchez
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Sánchez $\mathcal{P}_{i.o.}$ Valero

Representability

example (interval order)

FRIDAY 27 JUNE 2008 (Real 2)

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12:20-12:30	**B. Requejo** Dimension on topological spaces		**H.W. Martin** Lattices of Metrics on Cantor sets		**A. González** Biorthogonal systems in WCG Banach spaces
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Representability

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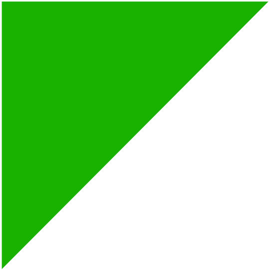

Representability

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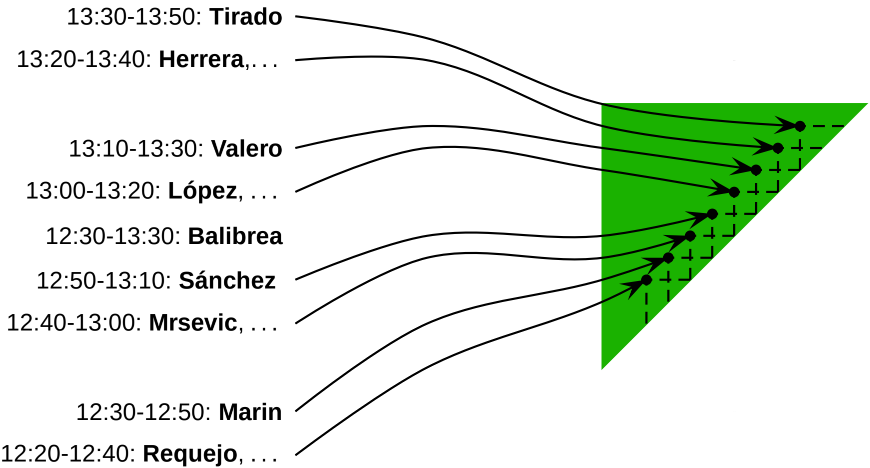

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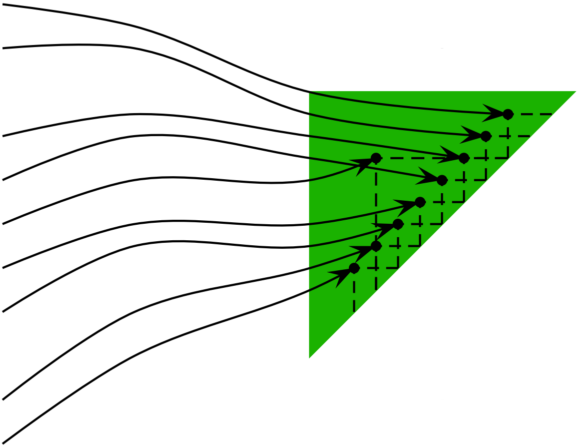
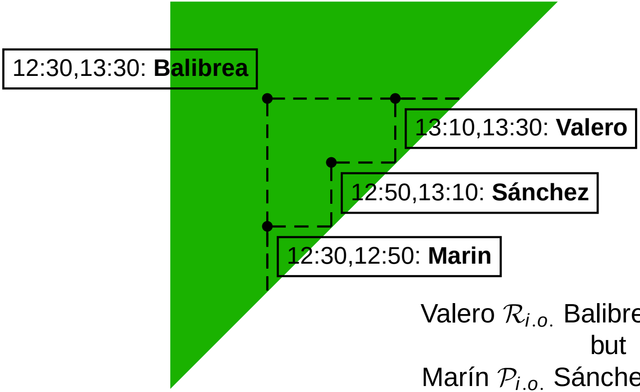


Representability

example (interval order)



Valero $\mathcal{R}_{i.o.}$ Balibrea $\mathcal{R}_{i.o.}$ Marín

but

Marín $\mathcal{P}_{i.o.}$ Sánchez $\mathcal{P}_{i.o.}$ Valero

Representability

equivalent formulation

Let (X, τ) a topological space. Then

(1) A total preorder $\mathcal{R}$ on (X, τ) is (continuously) representable if there exists a (continuous) $f : (X, \mathcal{R}, \tau) \rightarrow (\overline{\mathbb{R}}, \leq, \tau_u)$ such that $x\mathcal{R}y \iff f(x)\mathcal{R}_{t.p.}f(y) \;\; (x, y \in X)$.

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(2) A semiorder $\mathcal{R}$ on (X, τ) is (continuously) representable in $\overline{\mathbb{R}}$ if there exists a (continuous) $f : (X, \mathcal{R}, \tau) \to (\overline{\mathbb{R}}, \mathcal{R}_{s.o.}, \tau_u)$ such that $x\mathcal{R}y \iff f(x)\mathcal{R}_{s.o.}f(y) \ \ (x, y \in X)$.

WiAT '09 Workshop in Applied Topology

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(3) An interval order $\mathcal{R}$ on (X, τ) is (continuously) representable if there exists a (continuous) $f : (X, \mathcal{R}, \tau) \to (\mathcal{Y}, \mathcal{R}_{i.o.}, \tau_u)$ such that $x\mathcal{R}y \iff f(x)\mathcal{R}_{i.o.}f(y) \;\; (x, y \in X)$.

Representability

unified formulation

Note that all the results mentioned before are of the following form:

Let (X, τ) a topological space. Let $\mathcal{R}$ be a certain type of preference (total preorder, semiorder or interval order) on (X, τ) and $(L, \tau_L, \mathcal{R}_L)$ be the corresponding canonical structure. Then

The preference $\mathcal{R}$ on (X, τ) is continuously representable if there exists a continuous $f : (X, \tau, \mathcal{R}) \to (L, \tau_L, \mathcal{R}_L)$ such that

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The total preorder $\mathcal{R}$ on (X, τ) is continuously representable if there exists a continuous $f : (X, \tau, \mathcal{R}) \to (\overline{\mathbb{R}}, \tau_u, \leq)$ such that

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$$x\mathcal{R}y \iff f(x)\mathcal{R}_L f(y) \ (x, y \in X).$$

Notice that (L, τ_L) is always a completely distributive lattice and τ_L the Lawson topology on L.

Completely distributive lattices

Given a complete lattice L and $a, b \in L$, we write

$$a \blacktriangleleft b \iff \text{for each } A \subseteq L \text{ with } \bigwedge A \leq a, \text{ there is } c \in A \text{ with } c \leq b.$$

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For each $a \in L$ we write

$$U_{\leq}(a) = \{b \in L : a \leq b\}, \qquad L_{\leq}(a) = \{b \in L : b \leq a\},$$
$$U_{\blacktriangleleft}(a) = \{b \in L : a \blacktriangleleft b\}, \qquad L_{\blacktriangleleft}(a) = \{b \in L : b \blacktriangleleft a\}, \qquad \text{etc.}$$

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It is well-known that a lattice L is completely distributive if and only if

$$a = \bigwedge U_{\blacktriangleleft}(a) = \bigwedge \{b \in L : a \blacktriangleleft b\} \quad \text{for each } a \in L.$$

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Example: Any complete chain is completely distributive. In particular, if $L = \overline{\mathbb{R}}$, one just has $a \blacktriangleleft b \iff a < b$ for each $a, b \in \overline{\mathbb{R}}$. Hence

$$U_{\blacktriangleleft}(a) = U_{<}(a) = (a, +\infty] \quad \text{and} \quad L_{\blacktriangleleft}(a) = L_{<}(a) = [-\infty, a).$$

Completely distributive lattices

◀-separability

A subset $D \subseteq L$ is called meet-dense if each element $a \in L$ there exists some $D_a \subseteq D$ such that $a = \bigwedge D_a$.

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Examples: (1) $L = \overline{\mathbb{R}}$ is $\blacktriangleleft$-separable with $D = \mathbb{Q}$.

Completely distributive lattices

◀-separability

A subset $D \subseteq L$ is called meet-dense if each element $a \in L$ there exists some $D_a \subseteq D$ such that $a = \bigwedge D_a$.

A completely distributive lattice is said to be ◀-separable if it has a countable meet-dense subset.

Examples: (1) $L = \overline{\mathbb{R}}$ is ◀-separable with $D = \mathbb{Q}$.

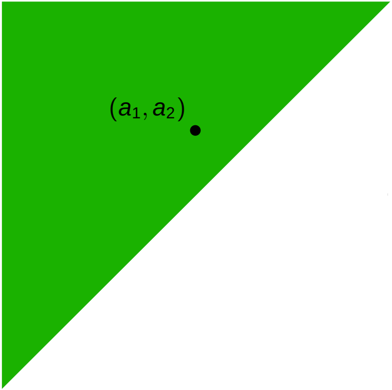
(2) $\mathcal{Y} = \{a \in \overline{\mathbb{R}}^2 : a_1 \leq a_2\}$ endowed with the componentwise order is a ◀-separable completely distributive lattice with

$$D = D_1 \cup D_2 = \{(q, q) \in \mathcal{Y} : q \in \mathbb{Q}\} \cup \{(q, +\infty) \in \mathcal{Y} : q \in \mathbb{Q}\}.$$

Completely distributive lattices

the canonical interval order

$$L = \mathcal{Y}$$



Completely distributive lattices

the canonical interval order

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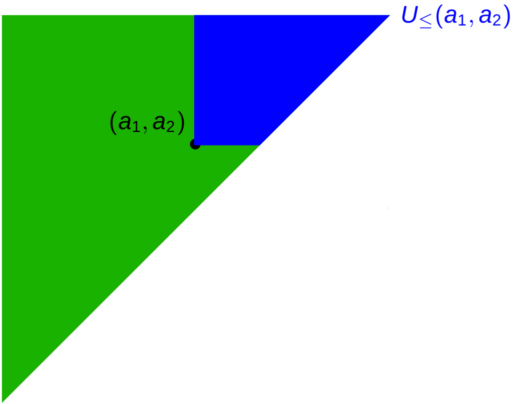

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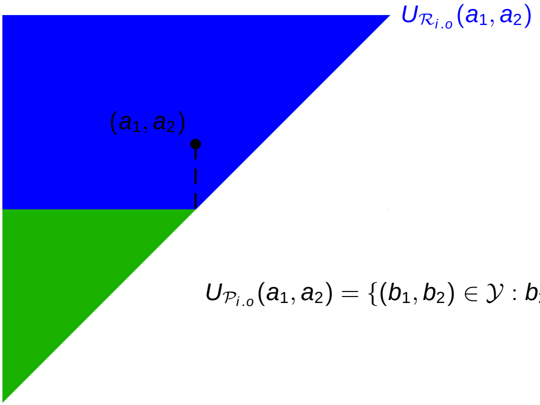

$$U_{\mathcal{P}_{i.o}}(a_1, a_2) = \{(b_1, b_2) \in \mathcal{Y} : b_2 \geq a_1\}$$

Completely distributive lattices

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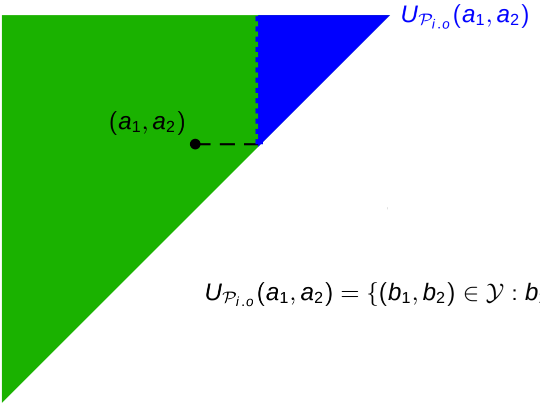

$$U_{\mathcal{P}_{i.o}}(a_1, a_2) = \{(b_1, b_2) \in \mathcal{Y} : b_1 > a_2\}$$

Completely distributive lattices

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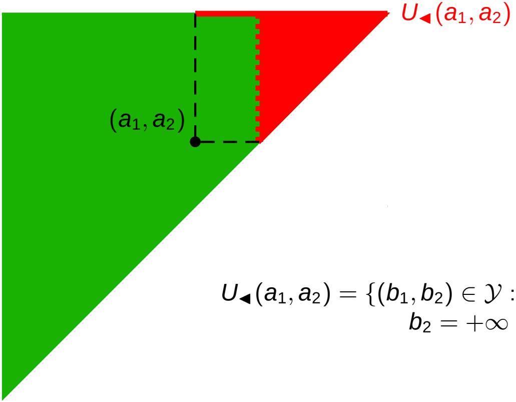

$$U_{\blacktriangleleft}(a_1, a_2) = \{(b_1, b_2) \in \mathcal{Y} : b_1 > a_2 \text{ or } b_2 = +\infty \text{ and } b_1 > a_1\}$$

Completely distributive lattices

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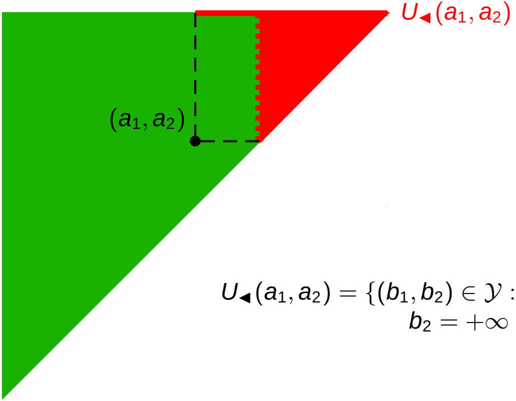

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$\forall (a_1, a_2) \in L \quad (a_1, a_2) = \bigwedge U_{\blacktriangleleft}(a_1, a_2) \implies L$ is completely distributive.

Completely distributive lattices

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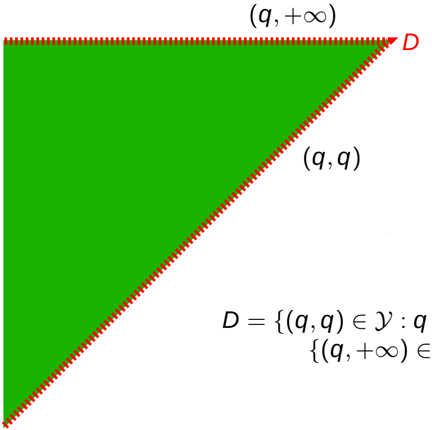

$$D = \{(q, q) \in \mathcal{Y} : q \in \mathbb{Q}\} \cup \{(q, +\infty) \in \mathcal{Y} : q \in \mathbb{Q}\}$$

Completely distributive lattices

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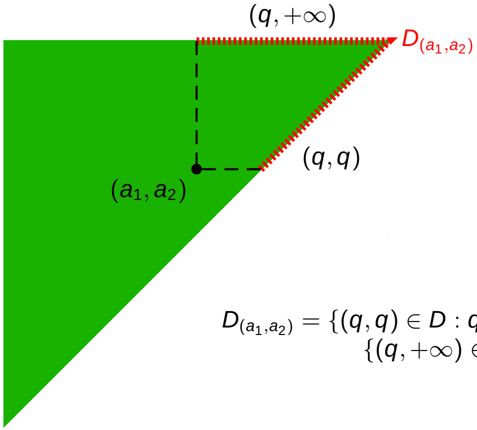

$$D_{(a_1,a_2)} = \{(q,q) \in D : q > a_2\} \cup \{(q,+\infty) \in D : q > a_1\}$$

Completely distributive lattices

the canonical interval order

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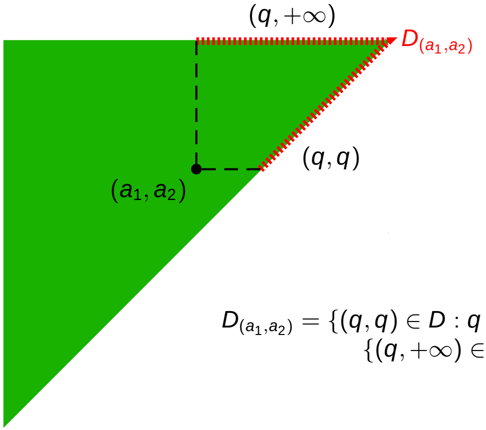

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$\forall (a_1, a_2) \in L \quad (a_1, a_2) = \bigwedge D_{(a_1,a_2)} \implies L$ is ◄-separable.

Completely distributive lattices

scales

Let X be a set, L completely distributive and $D \subseteq L$ meet-dense.

$\mathcal{F} = \{F_d \subseteq X : d \in D\}$ is said to be a $\blacktriangleleft$-*scale* if $\mathcal{F}$ is $\blacktriangleleft$-*increasing*, i.e.

$$F_{d_1} \subseteq F_{d_2} \text{ whenever } d_1 \blacktriangleleft d_2.$$

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The following are equivalent:

(1) $\mathcal{F}$ is a $\blacktriangleleft$-scale.

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If L is $\blacktriangleleft$-separable then we can choose D to be countable and so we conclude that we can identify L-valued functions on X with countable $\blacktriangleleft$-scales on X.

Completely distributive lattices

the Lawson topology

Any poset $(L, \leq)$ carries three well-known topologies:

- the *upper topology* $\nu(L)$ having $\{L \setminus L_{\leq}(a) : a \in L\}$ as a subbase.
- the *lower topology* $\omega(L)$ having $\{L \setminus U_{\leq}(a) : a \in L\}$ as a subbase.
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If L is a $\blacktriangleleft$-separable completely distributive lattice and $D \subseteq L$ a meet-dense subset. Then:

(1) $\{L \setminus L_{\leq}(d) : d \in D\}$ is a subbase of $\nu(L)$.

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WiAT '09 Workshop in Applied Topology

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Examples: (1) Let $L = \overline{\mathbb{R}}$ and $D = \mathbb{Q}$. Then $\{(q, +\infty] : q \in \mathbb{Q}\}$ and $\{[-\infty, q] : q \in \mathbb{Q}\}$ are, resp., subbases of $\nu(L)$ and $\nu(L)$.
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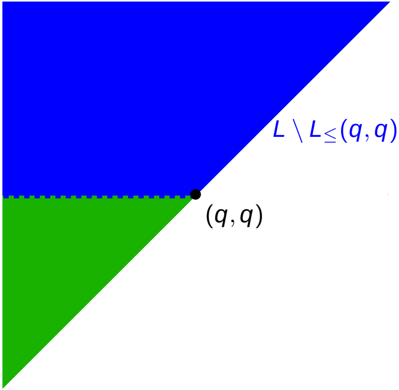

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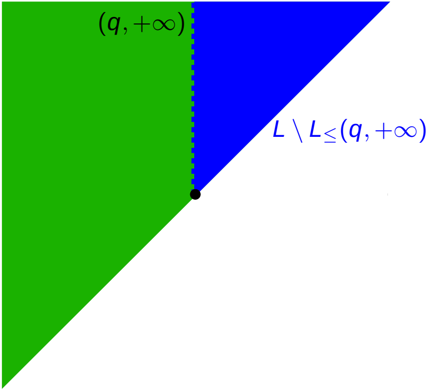

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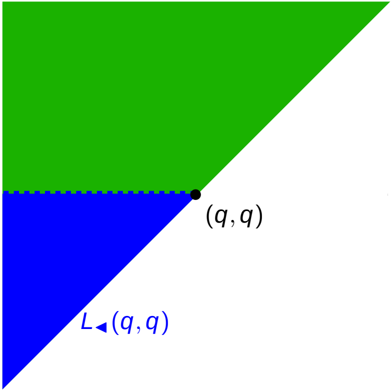

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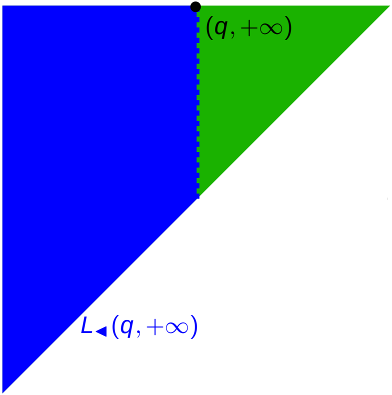

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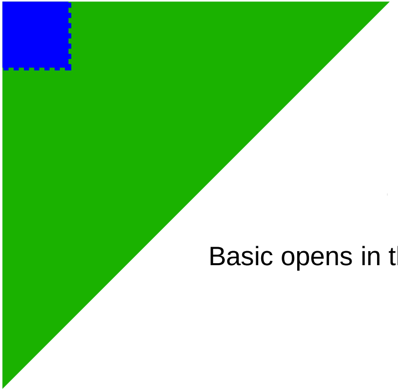

Basic opens in the Lawson topology

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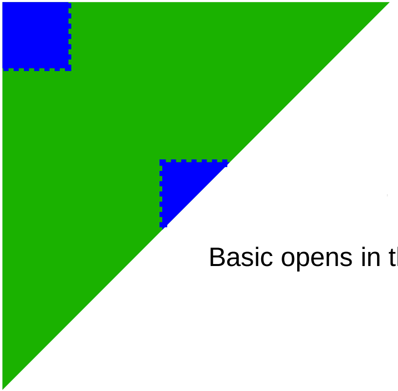

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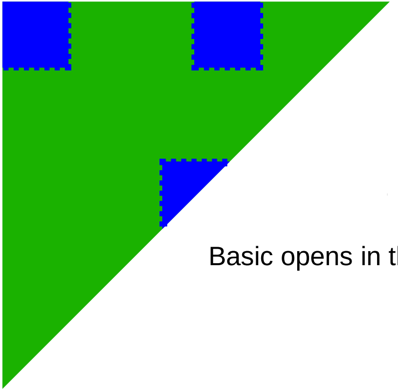

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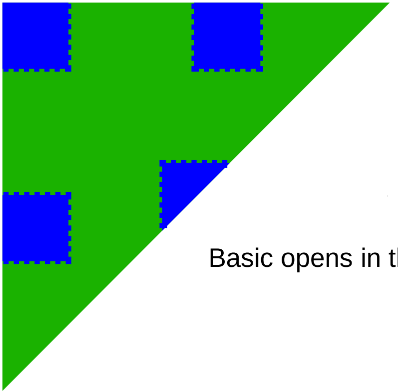

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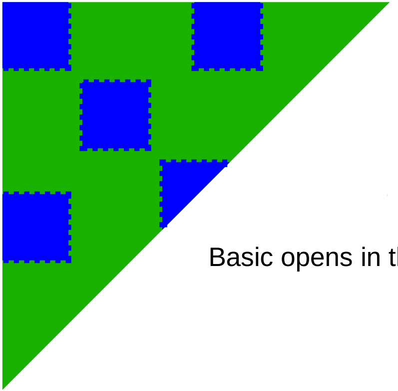

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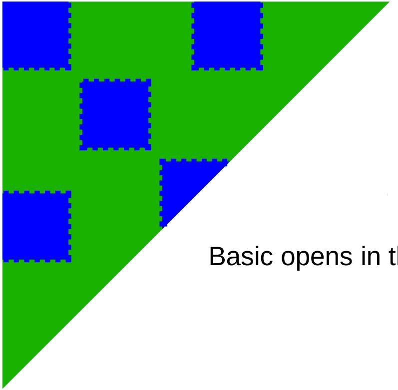

Basic opens in the Lawson topology

The Lawson topology is precisely the usual topology on $\mathcal{Y}$.

Completely distributive lattices

continuity

Given a topological space (X, τ) and $f : X \longrightarrow L$ we say that:

(1) f is *lower semicontinuous* iff it is continuous with respect to $\nu(L)$;

(2) f is *upper semicontinuous* iff it is continuous with respect to$\omega(L)$;

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After the equivalence stated between $\lhd$-scales on X and L-valued functions on X we have the following:

Theorem

Let $f : X \to L$ be generated by the $\lhd$-scale $\{F_d \subseteq X : d \in D\}$.

(1) *f is lower semicontinuous iff $\overline{F_{d_1}} \subseteq F_{d_2}$ whenever $d_1 \lhd d_2$;*

(2) *f is upper semicontinuous iff $F_{d_1} \subset \mathrm{Int}\, F_{d_2}$ whenever $d_1 \lhd d_2$;*

(3) *f is continuous iff $\overline{F_{d_1}} \subset \mathrm{Int}\, F_{d_2}$ whenever $d_1 \lhd d_2$.*

Main results

total preorders

A total preorder $\mathcal{R}$ on (X, τ) is *(continuously) representable* if there exists a (continuous) $u : (X, \tau, \mathcal{R}) \to (\mathbb{R}, \tau_u, \leq)$ such that

$$x\mathcal{R}y \iff u(x) \leq u(y) \ \ (x, y \in X).$$

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Then the following are equivalent:

(1) $\mathcal{R}$ *is representable;*

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A semiorder $\mathcal{R}$ on (X, τ) is *(continuously) representable in* $\overline{\mathbb{R}}$ if there exist a (continuous) $u : (X, \tau, \mathcal{R}) \to (\overline{\mathbb{R}}, \tau_u, \mathcal{R}_{s.o})$ such that

$$x\mathcal{R}y \iff u(x) \leq u(y) + 1 \ \ (x, y \in X).$$

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An interval order $\mathcal{R}$ on X is said to be *(continuously) representable* if there exists a pair of (continuous) $u, v : (X, \tau) \to (\mathbb{R}, \tau_u)$ such that

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