Light interference and PV cell design

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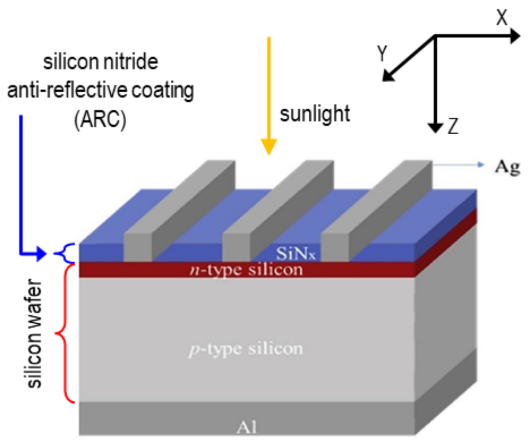

Figure 1: Schematic of a crystalline silicon photovoltaic solar (PV) cell.

Photovoltaic (PV) cells play a pivotal role in the research and development of clean-power technologies. At the simplest level, that is, leaving aside electronic, photonic, and optimal design aspects, PV cells constitute interesting physical systems to explore a series of problems involving wave optics, either at the level of electromagnetic optics or at the level of the scalar theory of light. In the problems below, we will considered a simplified model of a PV cell only consisting of a silicon nitride anti-reflective coating (ARC) layer stacked to a silicon wafer basis, as it is shown in Fig. 1 (any other element will be disregarded), with sunlight being incident perpendicularly on the ARC layer.

Part 1 (2 pts)

At a wavelength of 600 nm, the refractive indices for silicon nitride (ARC) and crystalline silicon (wafer basis) are $n_1 = 2.0439$ and $n_2 = 3.9310$, respectively. Moreover, the illuminated side of the ARC is in direct contact to air, with $n_0 \approx 1$. In order to elucidate whether the above value for the silicon nitride refractive index, n_1, makes of this material a suitable ARC, let us first determine the value that an ideal ARC should have and compare it with n_1. Thus, determine:

a) (1 pts) The value of the refractive index that maximizes the transmission across the ARC layer.

b) (1 pts) The value of the transmittance for both the refractive index obtained in part (a) and n_1.

Hint: At a dielectric-dielectric interface, under normal incidence conditions, the continuity of the tangential components of the electric and magnetic field amplitudes for the incident (i), reflected (r), and transmitted (t) waves satisfy the relations:

$$E_i + E_r = E_t, \quad H_i + H_r = H_t.$$

Note that scalars are used because each field is pointing along a given Cartesian coordinate.

Part 2 (3 pts)

The ARC forms a very thin layer staked to the silicon wafer in order to enhance transmission by interference. Neglecting multiple reflections inside the ARC layer, determine:

c) (1 pts) The phase condition that maximizes transmission across the ARC layer in terms of the wavelength of the incident light.

d) (1 pts) The thickness t of the ARC that warrants maximum transmission.

e) (1 pts) The thickness t beyond which the phase coherence gets lost and hence interference is expected not to have any longer effects on transmission.

Part 3 (1 pts)

A Michelson interferometer (see Fig. 2a) is used to determine with accuracy the thickness of the ARC layer. The interferometer is illuminated with a phosphor-based white LED, with power spectrum as displayed in Fig. 2b. Before proceeding, the interferometer must be properly aligned, which requires that both arms, L_1 and L_2, have exactly the same length.

f) (1 pts) Taking into account the spectral range covered by the power spectrum shown in Fig. 2b, obtain the coherence length associated with the LED source and determine the maximum path-length difference above which no interference is observed.

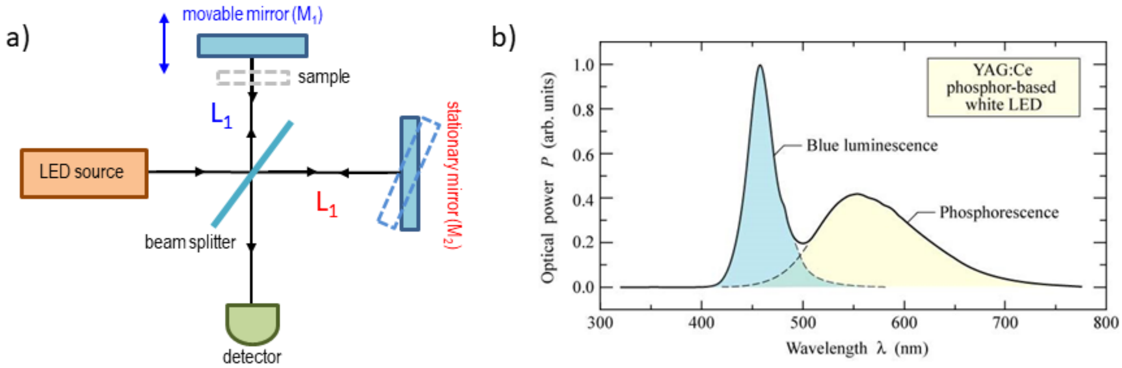

Figure 2: (a) Schematic of a Michelson interferometer and (b) spectrum of a phosphor-based white LED used to illuminate it.

Part 4 (2 pts)

To increase accuracy, the interferometric measurements are carried out in the spectral domain with the aid of a digital spectrometer (more reliable than taking them with the naked eye). To this end, the stationary mirror (M_2) is slightly tilted (blue dashed rectangle in Fig. 2a) until a single color hue is detected. When this color is analyzed with the spectrometer, a periodic fringed spectrum in the frequency domain arises (see Fig. 3), which cancels out if the length of both arms becomes equal. Taking this into account, determine:

g) (1 pts) The expression for the intensity distribution in the spectral domain and, from it, the separation between two consecutive frequencies for a given path-length difference (d) between the two arms.

h) (1 pts) The distance between two positions of the movable mirror (M_1) at which a spectral maximum occurs in the wavelength domain for the same wavelength λ.

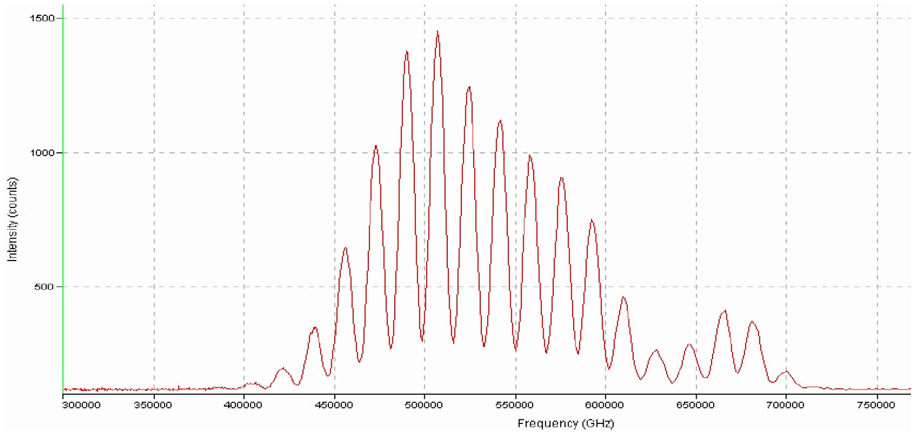

Figure 3: Power spectrum in the frequency domain measured with a digital spectrometer for a displacement d of the movable mirror M_1 with respect to the zero path-length position, using a white LED as light source.

Part 5 (2 pts)

A single ARC layer is too thin to perform measurements with the interferometer, so 20 of them are stacked together. This sample is then inserted in the interferometer, at an intermediate position along the arm L_1 and perpendicular with respect to the light pathway, as shown in Fig. 2a (see gray dashed rectangle). Determine:

i) (1 pts) The expression that relates the thickness t of a transparent sample with the displacement d of the movable mirror.

j) (1 pts) The thickness of a single ARC layer if the full sample generates a displacement s (with respect to the zero path-length difference), such that the maxima of the power spectrum have a periodicity of 98 THz in the frequency domain.

Solutions to Part 1

a) (1 pts) The value of the refractive index that maximizes the transmission across the ARC layer.

In order to determine the optimal value for the coating refractive index, first it is necessary to determine the transmittance at the coating-silicon interface, and then compute the value of the refractive index that maximizes this quantity. Since the materials involved (wafer and anti-reflective coating) are dispersive, the problem can be solved appealing to a monochromatic plane-wave approach. Accordingly, consider a frequency ω. At the interface between two dielectrics, at normal incidence conditions, the continuity of the tangential components of the electric and magnetic field amplitudes of the incident (i), reflected (r), and transmitted (t) waves satisfy the equations:

$$E_i + E_r = E_t, \tag{1}$$
$$H_i + H_r = H_t, \tag{2}$$

where scalars are used because each field is pointing along a given Cartesian coordinate (assuming the wave vector points downwards, from the first to the second medium). Also, from the transversality condition between the two fields, it is easily shown that their amplitudes satisfy the relationship:

$$H = \frac{kE}{\mu_0 \omega}. \tag{3}$$

If $\vec{k}_i = (0,0,k_i)$, $\vec{k}_r = (0,0,-k_i)$, and $\vec{k}_t = (0,0,k_t)$, Eq. (2) reads as

$$k_i E_i - k_i E_r = k_t E_t. \tag{4}$$

From this equation and Eq. (1) we readily obtain the reflection coefficient:

$$r = \frac{E_r}{E_i} = \frac{k_i - k_t}{k_i + k_t}. \tag{5}$$

Within the simplistic model here considered (disregarding absorption with the penetration distance), the transmittance is given by:

$$T = 1 - R = 1 - |r|^2 = \frac{4k_i k_t}{(k_i + k_t)^2}, \tag{6}$$

which can alternatively be recast, in terms of the corresponding refractive indices, as:

$$T = \frac{4n_i n_t}{(n_i + n_t)^2}. \tag{7}$$

In the problem here, we have two interfaces, namely, air-coating (01) and coating-silicon (12), so the total transmittance reads as:

$$T = T_{01} T_{12} = \frac{16 n_0 n_1^2 n_2}{(n_0 + n_1)^2 (n_1 + n_2)^2}. \tag{8}$$

Maximum absorption is achieved when there is maximum transmission through the coating medium (minimum reflection), since the amount of available energy to be harvested by the silicon wafer will be larger. We thus need to find the value of n_1 that maximizes T. Solving the equation

$$\frac{\partial T}{\partial n_1} = 0, \tag{9}$$

we find

$$n_1 = \sqrt{n_0 n_2} = 1.9827. \tag{10}$$

According to this simple relation, we obtain

$$\boxed{n_1 = 1.9827} \tag{11}$$

which is pretty close to the refractive index of the silicon nitride (2.0439), with a deviation of about 3%.

It can readily be shown that the above value maximizes the transmittance after computing the second derivative of the latter:

$$\frac{\partial^2 T}{\partial n_1^2} = -\frac{16}{n_0^2 n_2^2} \frac{\left(\sqrt{\frac{n_0}{n_2}} + \sqrt{\frac{n_2}{n_0}} - 1\right)}{\left(1 + \sqrt{\frac{n_0}{n_2}}\right)^2 \left(1 + \sqrt{\frac{n_2}{n_0}}\right)^4} < 0. \tag{12}$$

b) (1 pts) The value of the transmittance for the refractive index obtained in part (a)

Substituting the value for n_1 found in part (a) into Eq. (8), we obtain

$$T = \frac{16 n_0 n_2}{\left(\sqrt{n_0} + \sqrt{n_2}\right)^4} = \frac{4 n_2}{n_0} \frac{1}{\left(1 + \sqrt{\frac{n_2}{n_0}}\right)^4}. \tag{13}$$

Since $n_0 = 1$, this expression reduces to:

$$T = \frac{16 n_2}{\left(1 + \sqrt{n_2}\right)^4}, \tag{14}$$

which takes the value:

$$\boxed{T = 0.7935} \tag{15}$$

It can be noted that the value of the transmittance has increased up to 0.7935 from 0.6450, when the anti-reflective coating is absent, which means a decrease of reflectivity of about 20%. If instead of the optimal value of the refractive index we consider the value for the silicon nitride, the transmittance is 0.7932, which is basically the same value obtained with the optimal refractive index.

Solutions to Part 2

c) (1 pts) The phase condition that maximizes transmission across the ARC layer in terms of the wavelength of the incident light.

In this case, the main phenomenon that takes place is interference, which arises from the superposition between the wave reflected off the air-coating interface and the wave that enters the coating layer, and then is reflected backwards at the coating-silicon interface. Multiple reflections inside the coating layer are neglected. Since we are interested in maximum transmission at the wavelength considered, there must be minimum (vanishing) reflection, which is achieved by choosing the thickness of the coating layer in such a way that the two waves mentioned before interfere destructively.

Regardless of the corresponding amplitudes, since $n_2 > n_1 > n_0$, there are no π-phase changes accumulated, and the phase difference carried by the second wave with respect to the former is:

$$\Delta\phi = 2k_1 t = \frac{4\pi n_1 t}{\lambda}. \tag{16}$$

The condition for destructive interference is that this phase equals an odd multiple of π, which is equivalent to having the distance traveled inside the coating layer (i.e., twice its thickness) equal to an odd multiple of half wavelengths $\lambda_1 = \lambda/n_1$:

$$\boxed{2t = \left(m + \frac{1}{2}\right)\lambda_1} \tag{17}$$

d) (1 pts) The thickness t of the ARC that warrants maximum transmission.

Following part (c), we find that the expression for the optimal or minimum thickness is

$$t = \frac{\lambda}{4n_1} \tag{18}$$

Taking into account the values of the wavelength and the refractive index for the silicon nitride, the minimum thickness that ensures maximum cancellation of the reflected way (maximum anti-reflective coating) is

$$\boxed{t = 73.389 \text{ nm}} \tag{19}$$

Other values are $t = 220.17$ nm $(m = 1)$, 366.95 nm $(m = 1)$, 513.72 nm $(m = 1)$, etc., following Eq. (17).

e) (1 pts) The thickness t beyond which the phase coherence gets lost and hence interference is expected not to have any longer effects on transmission.

Although a series of optimal interference-mediated thickness values arises increasing m in (17), note that the photovoltaic cell is illuminated by the full solar spectrum, which in the visible range goes from the 400 to the 790 THz, approximately, and hence it has a rather limited coherence length. In particular, if we consider the visible electromagnetic spectral range $\Delta f = 390$ THz, this renders a coherence time of $t_c \approx 1/\Delta f \approx 2.56$ fs, and hence a coherence length:

$$\ell_c = \frac{ct_c}{n_1} \approx 375.75 \text{ nm}. \tag{20}$$

This value determines the range above which phase coherence is lost with non-monochromatic light and therefore it is expected to loose the benefit provided by interference so far. Thus, in the current case, the thickness of the thickest coating layer is

$$\boxed{t \approx 366.95 \text{ nm}} \tag{21}$$

Solutions to Part 3

f) (1 pts) Taking into account the spectral range covered by the power spectrum shown in Fig. 2b, obtain the coherence length associated with the LED source and determine the maximum path-length difference above which no interference is observed.

From the figure, it is seen that the spectrum of the LED source approximately covers the range that goes from 420 nm to 730 nm, i.e., from the 710 THz to the 410 THz. From Eq. (20), we have a coherence length

$$\boxed{\ell_c = ct_c \approx 1 \ \mu\text{m}} \tag{22}$$

If we take into account that the path-length difference between the two arms is larger than this quantity, then the waves traveling along each arm will not interference.

Solutions to Part 4

g) (1 pts) The expression for the intensity distribution in the spectral domain and, from it, the separation between two consecutive frequencies for a given path-length difference (d) between the two arms.

At some point beyond the beam-splitter, once the two waves are recombined, appealing to the usual scalar theory of light, we can represent the total amplitude as:

$$\Psi = \psi_1 + \psi_2 \sim A_1 + A_2 e^{2ikd+i\pi} \tag{23}$$

where A_1 and A_2 denote the amplitudes of each wave ("1" for the wave traveling towards the fixed mirror and "2" for the wave reaching the movable mirror), which might depend on the position, although both will be pretty similar if the interferometer is properly aligned. As it is seen, the only difference between the two partial waves is the phase factor associated with the path length difference, $2d$, where d is how much the movable mirror has been displaced with respect to the zero path-length position, plus a π factor arising from an extra reflection. Therefore, the corresponding intensity measured (or power, depending on the kind of detector used), the first answer to the question, is going to be proportional to the quantity:

$$\boxed{|\Psi|^2 \sim |A_1|^2 + |A_2|^2 - |A_1||A_2|\cos(2kd) = |A_1|^2 + |A_2|^2 - |A_1||A_2|\cos\left(\frac{4\pi d}{\lambda}\right)} \quad (24)$$

Note from Eq. (24) that, for a given displacement d of the movable mirror, there is a set of wavelengths that produce maxima and minima, although they are not evenly spaced. However, if the spectrum is inspected in terms of the frequency, we find that maxima, for instance, satisfy the condition

$$f_m = \left(m + \frac{1}{2}\right)\frac{c}{2d}. \quad (25)$$

so that the distance between two consecutive maxima (or minima, changing the condition) is constant, since

$$\boxed{\Delta f = f_{m+1} - f_m = \frac{c}{2d}} \quad (26)$$

i.e., they are evenly spaced, which allows to determine the displacement d (with respect to the zero path-length position) straightforwardly.

h) (1 pts) The distance between two positions of the movable mirror (M_1) at which a spectral maximum occurs in the wavelength domain for the same wavelength λ.

Now, we need reconsider the phase factor in Eq. (refeq20) keeping λ fixed and finding two consecutive values of d that make maximum this interference term. The maximum condition is similar to the one given by Eq. (25), but referred to these displacements and the fixed frequency, i.e.:

$$\frac{4\pi d}{\lambda} = (2m+1)\,\pi. \quad (27)$$

We are not sure which exact interferential order m is associated with the spectrum maximum for the selected wavelength, observed for a displacement, say, d_a. However, if we do know that by increasing the displacement, denoted as d_b, we shall reach the next order, $m+1$. Therefore, from Eq. (27) we obtain that the distance between two consecutive positions where a spectral maximum is observed for a specific wavelength λ is:

$$\boxed{d_b - d_a = \frac{\lambda}{2}} \quad (28)$$

which is also a constant (for a given λ), that is, twice the resolution limit for a given wavelength (which determines the distance between a maximum and the next minimum).

Solutions to Part 5

i) (1 pts) The expression that relates the thickness t of a transparent sample with the displacement d of the movable mirror.

Introducing a medium with a different refractive index is equivalent to changing the optical path length. In this case, if the phase difference introduced with the replacement of a length equivalent to the thickness of the material, $\varphi = 2kt - 2k_0t = 4\pi(n-1)t/\lambda$ (k and k_0 make reference to the wavenumber with and without the material), equals the corresponding distance, $2k_0d$, required to reach the zero path-length condition, which renders the expression:

$$\boxed{d = (n-1)t} \tag{29}$$

j) (1 pts) The thickness of a single ARC layer if the full sample generates a displacement s (with respect to the zero path-length difference), such that the maxima of the power spectrum have a periodicity of 98 THz in the frequency domain.

Taking into account that a displacement d produces an spectral interference diagram with period Δf in the frequencies, from Eqs. (26) and (29), and taking into account that we have 20 coating layers, we find

$$t = \frac{1}{20}\frac{c}{2(n-1)\Delta f}, \tag{30}$$

which, for $\Delta f = 98$ THz, renders the value

$$\boxed{t = 73.312 \text{ nm}} \tag{31}$$

Note that this value is pretty close to the value estimated in part (e).