WEIGHTED INEQUALITIES FOR THE HARDY-LITTLEWOOD MAXIMAL OPERATOR

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Abstract: In this talk we will deal with weighted inequalities for the Hardy Littlewood maximal operator, M. Recall that:

(i) The boundedness of M on Lebesgue spaces $L^p(u)$ is characterized by the A_p Muckenhoupt class of weights.
(ii) The boundedness of M on classical Lorentz spaces $\Lambda^p(w)$ is solved by the B_p class of weights.

Whereas the first problem is solved using techniques from the Calderón-Zygmund theory, the second one requires tools from the theory of rearrangement invariant spaces [5].

The so-called weighted Lorentz spaces $\Lambda^p_u(w)$, introduced in [7], generalize $L^p(u)$ and $\Lambda^p(w)$ spaces, providing a common framework to study (i) and (ii). The strong-type inequality for M on $\Lambda^p_u(w)$ was solved in [4] and [6]. However, the weak-type inequality for $\Lambda^p_u(w)$ remained open.

The goal of this talk is to discuss the solution to this problem, obtained in [1]. Then, we will relate our results with weighted inequalities for the Hilbert transform. Our techniques are different from the classical cases and so they also provide alternative proofs to the known results. This talk is based on joint works with J. Antezana, M. J. Carro, and J. Soria (see [1], [2] and [3]).

References

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